Additional problems, week 1-4

Example 1. *Compute the solution of the pde*

$$yz_x - xz_y = 0.$$

Example 2. *Compute the solution of the pde*

$$z_x + yz_y = 0.$$

Example 3. *Compute the solution of the pde*

$$\sqrt{1-x^2}z_x + z_y = 0, \qquad z(0,y) = y.$$

Example 4. *Find the integral surfaces of the vector field* $(x^2, y^2, (x+y)z)^T$ *containing the line*

(a) $x = y = z$,

(b) $x = 1$, $y = z$,

(c) $x = y$, $z = x^2$.

Example 5. *Compute the solution of the pde* $zu_x + yu_y + xu_z = 0$.

Example 6. *Compute the solution of the pde* $x^2u_x + y^2u_y + (x+y)zu_z = 0$.

Example 7. *Solve the pdes*

(a) $az_x + bz_y = -cz$,

(b) $xz_x + (y-1)z_y = xz$.

Example 8. *Solve the initial value problem*

$$xz_x + (y-1)z_y = xz$$
$$z(x,0) = g(x).$$

Example 9. *Find the general solution* $u(x,y,z)$ *of the equation*

$$xu_x + (y-1)u_y + xzu_z = 0.$$

Additional problems, week 5

Example 10. *Classify* $u_{xx} + u_{xy} + u_{yy} = u_x + u_y + u$ *as elliptic, parabolic or hyperbolic.*

Example 11. *Classify* $8u_{xx} + 6u_{yy} + 4u_{zz} + u_{xy} + 2u_{xz} + u_{yz} = 0$ *as elliptic, parabolic or hyperbolic.*

Example 12. *Classify* $2u_{xy} - 2u_{xz} + 2u_{yz} + 3u_x - u = 0$ *as elliptic, parabolic or hyperbolic.*

Example 13. *Compute the principal part and solve the characterictic equation for*

(a) $u_{xx} - 4u_{xy} + 4u_{yy} + 2u_y + u = 0$,

(b) $u_{xx} + 2u_{xy} - 3u_{yy} + 3u_x - u = 0$,

(c) $\mathbf{e}^{2y} u_{xx} - \mathbf{e}^{2x} u_y y = 0$

Example 14. *Transform the pde*

$$u_{xx} + 2u_{xz} + u_{yy} + 2u_{yz} + 2u_{zz} = 0$$

into the canonical form.

Additional problems, week 6

Example 15. *Solve the wave equation for* $u(x,0) = \mathbf{e}^x$, $u_t(x,0) = \sin(x)$.

Example 16. *Solve the wave equation for* $u(x,0) = \ln(1+x^2)$, $u_t(x,0) = x - 4$.

Example 17. *Solve the wave equation* $u_{tt} - u_{xx} = 0$ *for* $0 < x < 1$, $t > 0$ *for the initial conditions* $u(x,0) = \sin(\pi x)$, $u_t(x,0) = \sin(2\pi x)$.

Example 18. *Solve the wave equation*

$$
\begin{aligned}
&u_{tt} - \Delta u = 0,\ t > 0,\\
&u(x,0) = 0,\\
&u_t(x,0) = p(x).
\end{aligned}
$$

for $x \in \mathbb{R}^3$ *with* $p(x_1,x_2,x_3) = x_1 x_2$ *using Kirchhoff's formula.*

Additional problems, week 10

Example 19. *Compute the solution for a circle with radius 1 and the boundary condition* $g(\varphi) = \pi^2 - \varphi^2$ *for* $-\pi \leq \varphi \leq \pi$.

Example 20. *Compute the solution for a circle with radius 1 and the boundary condition* $g(x, y) = x^2 - y^2 - x$.