London Mathematical Finance Group

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2016-17 London Mathematical Finance Seminar Series

The **January-March 2017** programme is hosted by Cass Business School, City, University of London.

Date: Thursday 26 January 2017

Speaker #1: **Ivar Ekeland**, CEREMADE and Institut de Finance, Université Paris-Dauphine

Time: 18:15-19:00

Place: Cass Business School, 106 Bunhill Row, Room 6001 (6th floor)

Find out more about Ivar Ekeland here

Storers, processors and speculators: an equilibrium model for commodity markets

Date: Thursday 6 April 2017

Speaker #1: **Peter Takáč**, Mathematics, Universität Rostock

Time: 18:15-19:00

Place: Cass Business School, 106 Bunhill Row, Room 6001 (6th floor)

Find out more about Peter Takáč here

Stochastic Volatility Models, Complete Markets, and Analyticity of Solutions to Ito's Parabolic Problems

The problem of constant volatility, $\sigma > 0$, in the Black-Scholes option pricing model has sparked a number of new research directions on the nature of volatility. We first briefly recall the "most popular" volatility models based on mean reversion (the Ornstein-Uhlenbeck process): (i) Heston's stochastic volatility model (1993); (ii) an alternative, mathematically much "easier" model due to Fouque, Papanicolaou, and Sircar (2000); and (iii) a more sophisticated model due to Dupire (1992) based on local implied volatility. We will discuss Dupire's model and a possible generalization to volatility depending also on the asset (stock) price (as suggested by Lewis (2000)). This generalization does not cause any new mathematical difficulty (from an analytic or probabilistic point of view; numerical implementation might be harder).

Although Heston's model for option pricing (i) is the simpliest model with stochastic volatility (SV), its rigorous analytical treatment is quite involved due to the degeneracies in Ito's parabolic problem for very low and very high volatility levels (2016). This analysis is motivated by the analytical treatment of the alternative SV model (ii) due to Fouque, Papanicolaou, and Sircar which is much "easier" to handle. Ito's parabolic problem for this model turns out to be uniformly parabolic with bounded analytic coefficients. Hence, it is not surprising that the solution is analytic in both, space and time variables, even if the initial data are only continuous. The "space" variable stands for the pair of the asset price and the volatility. We will prove the analyticity result for the option price by standard L^2-methods in the Hardy space H^2 of holomorphic functions that extend the real analytic functions to a suitable complex parabolic domain (2012).
We finish our lecture by applying well-known results on complete markets by M. H. A. Davis and J. Obloj (2008) to model (ii): Analyticity result for the option price implies that this option completes the market.