

On the Strong Comparison Principle for Degenerate Elliptic Problems with Convection.

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Abstract

We will discuss the weak and strong comparison principles (**WCP** and **SCP**, respectively) for quasilinear elliptic boundary value problems with the p -Laplacian in one space dimension, $\Delta_p(u) \stackrel{\text{def}}{=} \frac{d}{dx} (|u'|^{p-2}u')$. We treat the “degenerate” case of $2 < p < \infty$ and allow also for the nontrivial **convection velocity** $b : [-1, 1] \rightarrow \mathbb{R}$ in the underlying domain $\Omega = (-1, 1)$. We establish the **WCP** under a rather general, “natural sufficient condition” on the convection velocity, $b(x)$, and the reaction function, $\varphi(x, u)$. Furthermore, we establish also the **SCP** under a number of various additional hypotheses. In contrast, with these hypotheses being violated, we present also a few rather natural counterexamples to the **SCP** and discuss their applications to an interesting classical problem of fluid flow in porous medium, “*seepage flow of fluids in inclined bed*”. Our methods are based on a mixture of classical and new techniques.

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