



# Geometrische Objekte in höheren Dimensionen

Was ist das und was kann man damit machen?

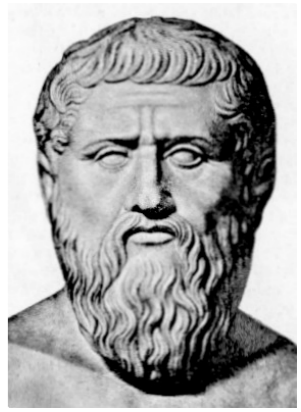
Prof. Dr. Achill Schürmann  
(Institut für Mathematik / Geometrie)

<http://www.geometrie.uni-rostock.de/>

# Geometrische Objekte in 3D

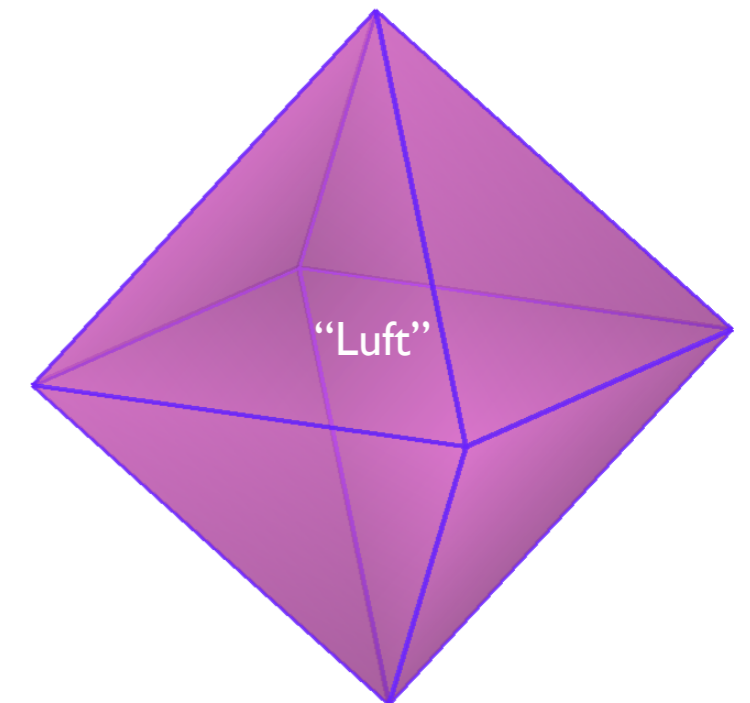
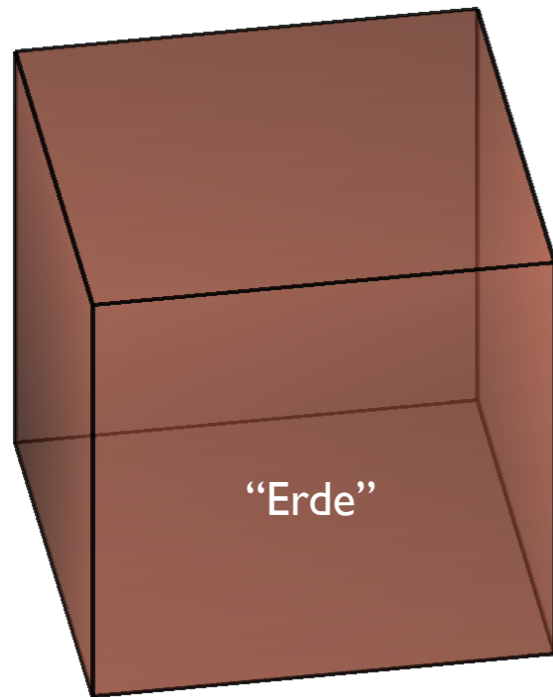
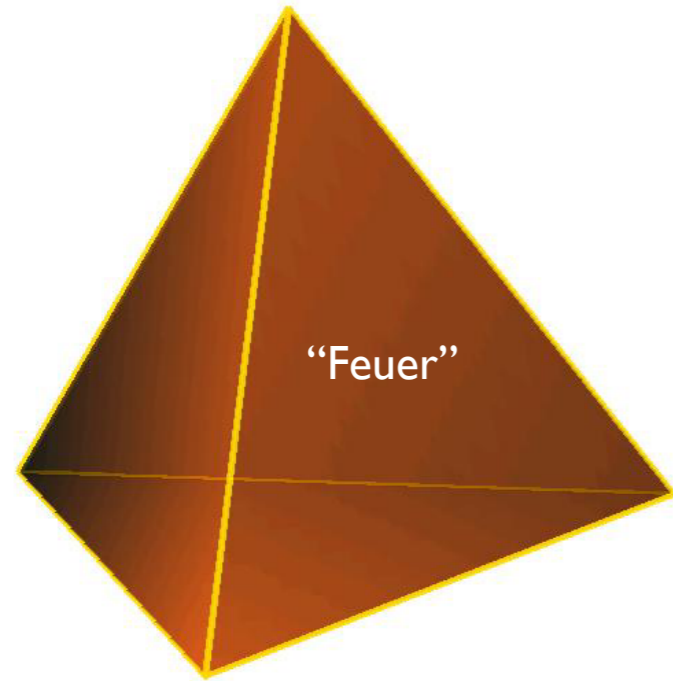
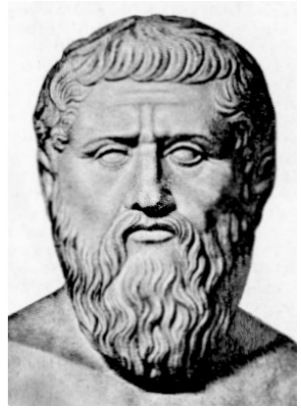


# Platons Timaios (~360 v. Chr)



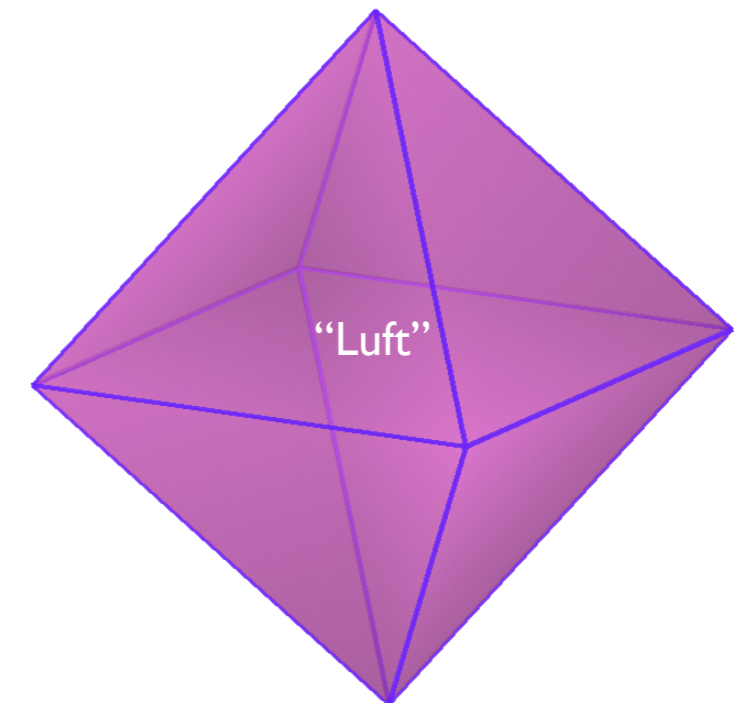
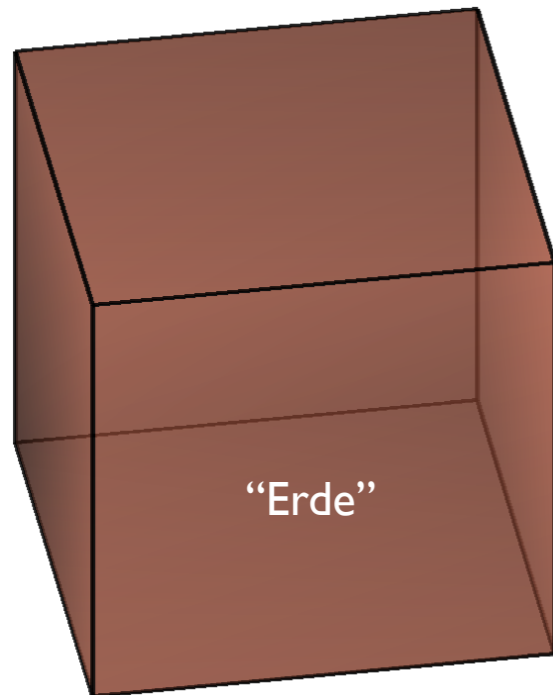
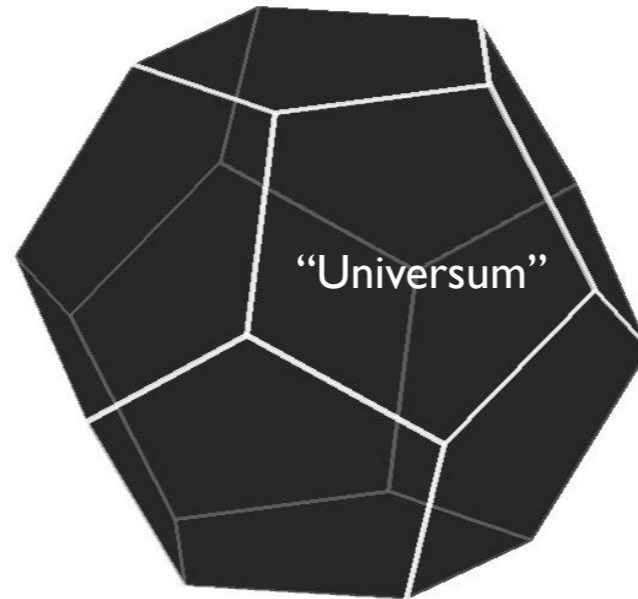
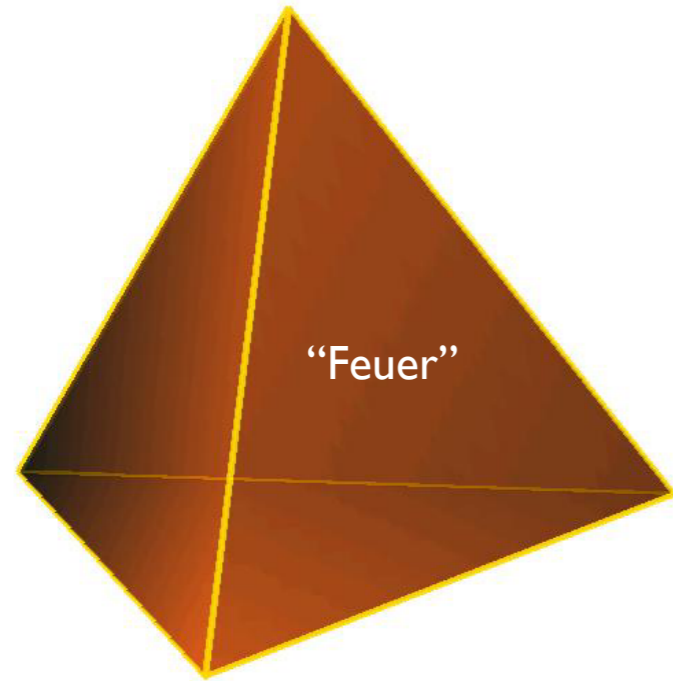
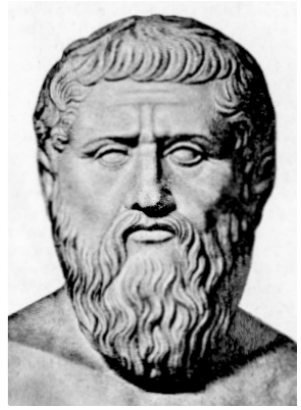
Aus Raffael's "Die Schule Athens" (1510/11)

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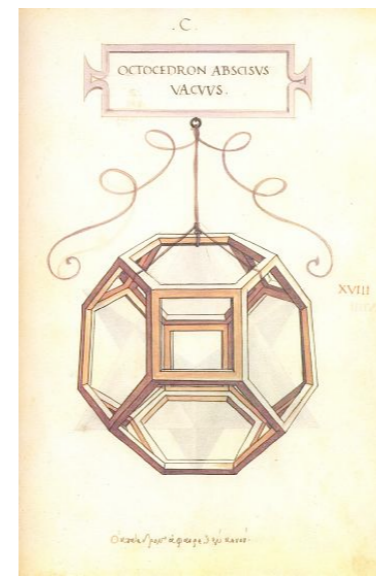
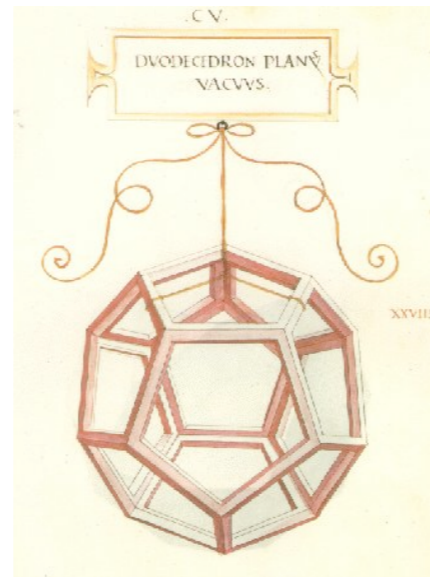
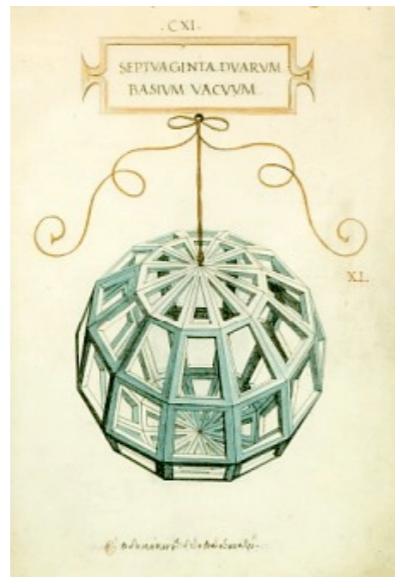
# Platons Timaios (~360 v. Chr)



Aus Raffael's "Die Schule Athens" (1510/11)

# da Vinci, Dürer, Kepler, ...

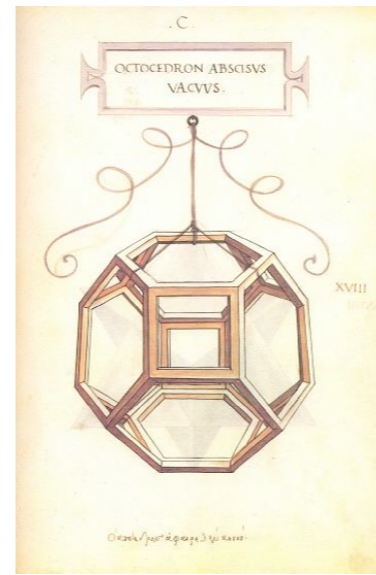
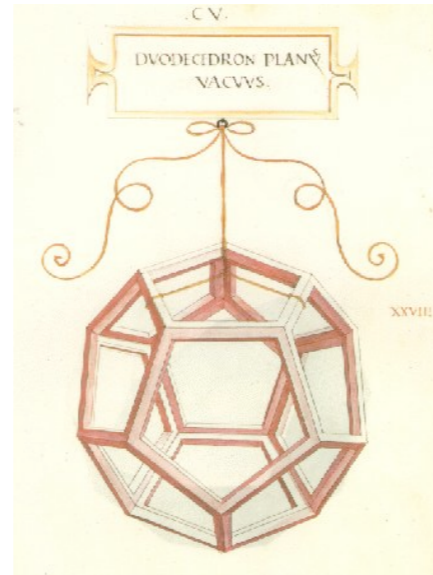
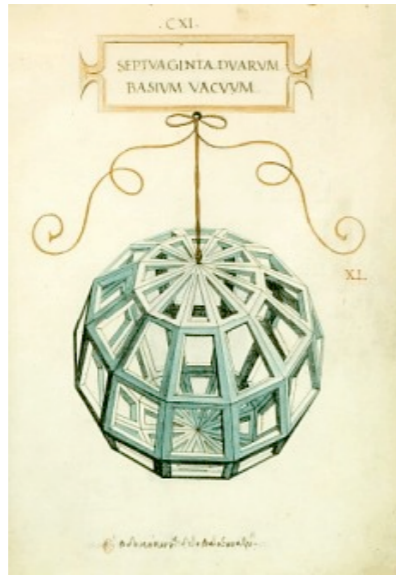
## Die Renaissance der Polyeder



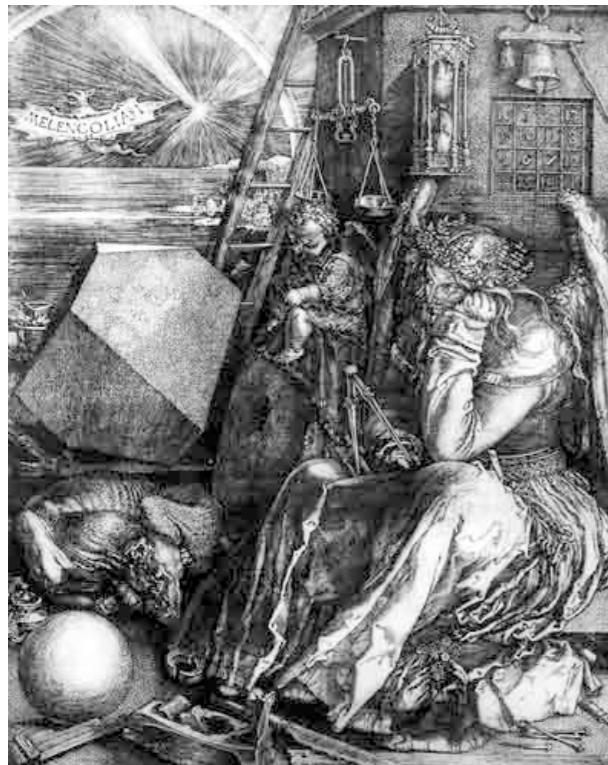
Leonardo da Vinci  
(1452-1519)

# da Vinci, Dürer, Kepler, ...

## Die Renaissance der Polyeder



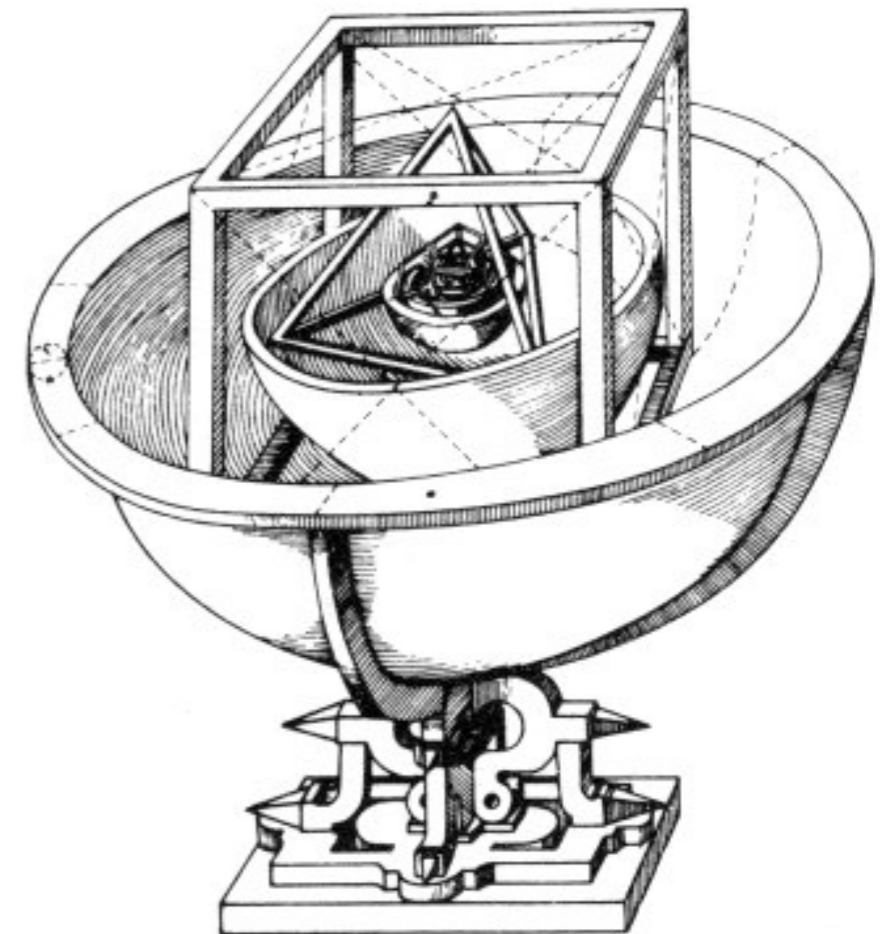
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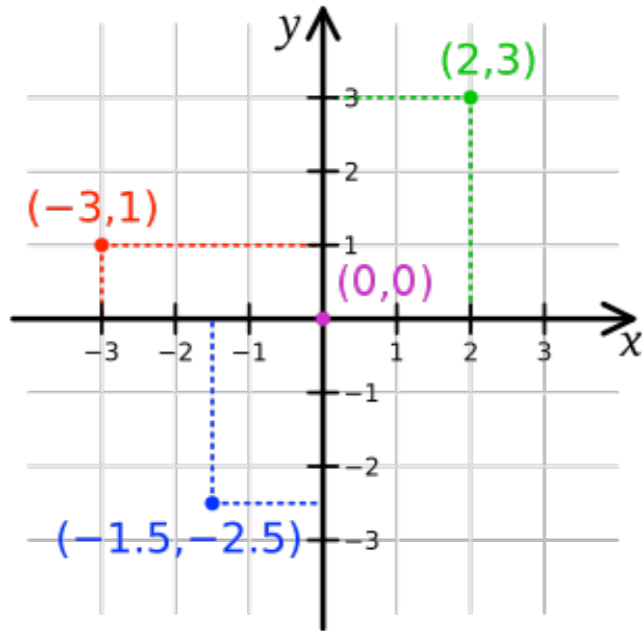
Albrecht Dürer  
(1471-1528)



Johannes Kepler  
(1571-1630)



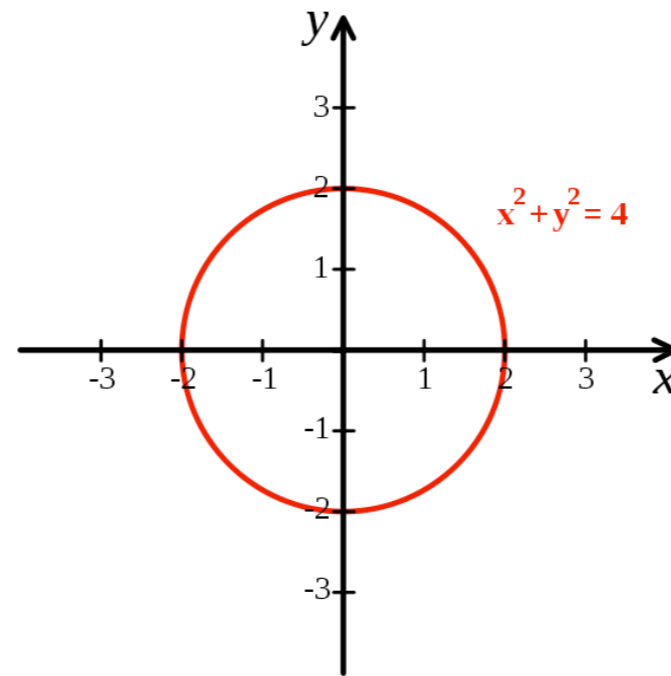
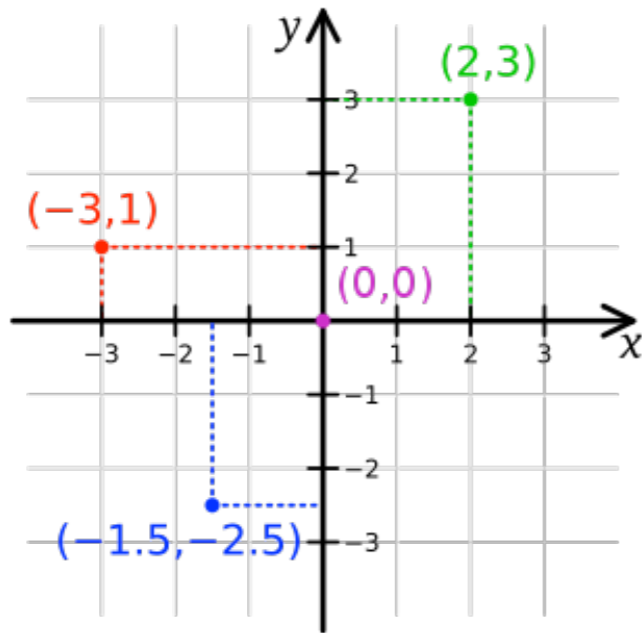
# Kartesische Koordinaten



René Descartes  
(1596-1650)

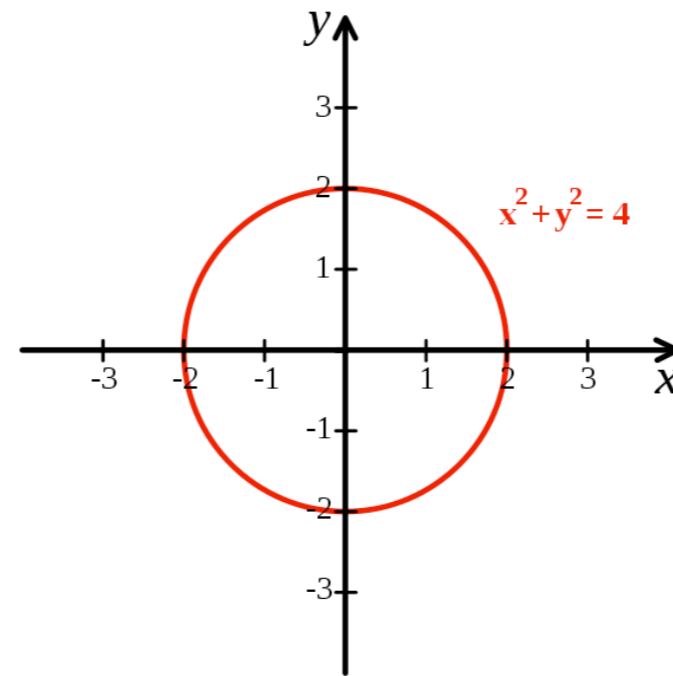
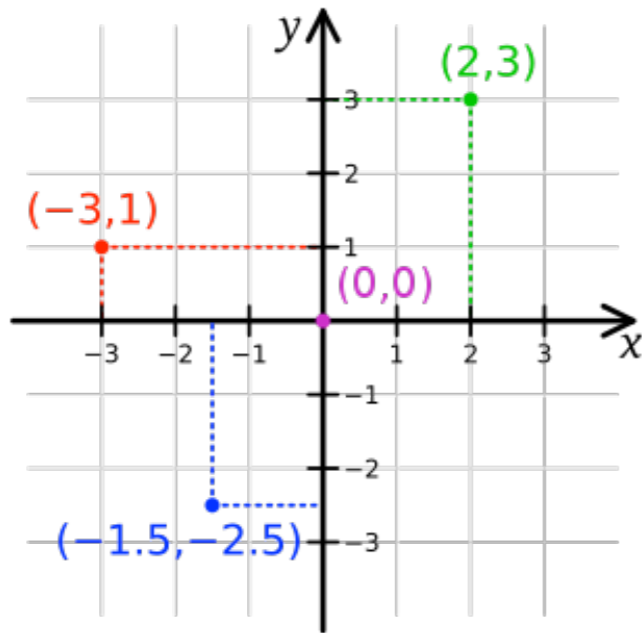


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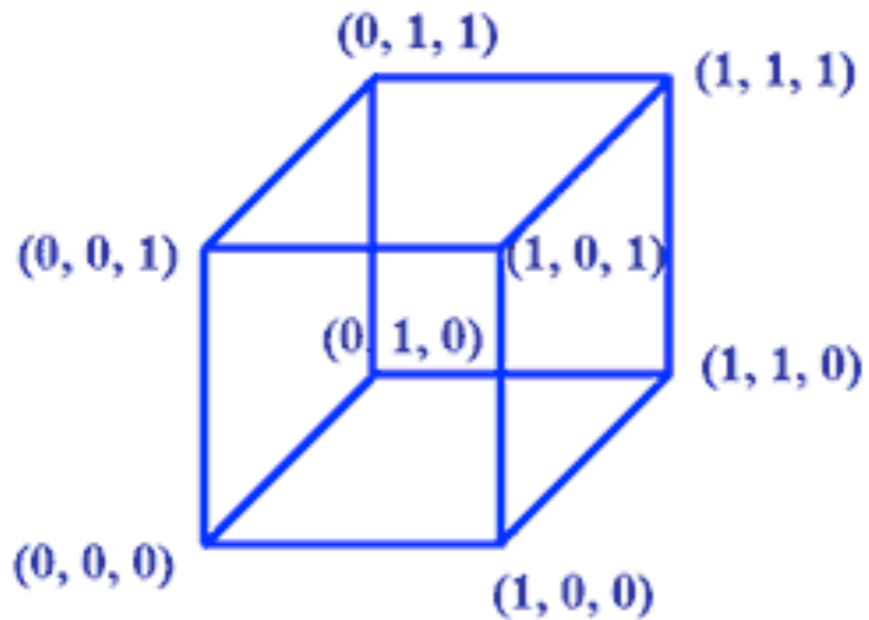


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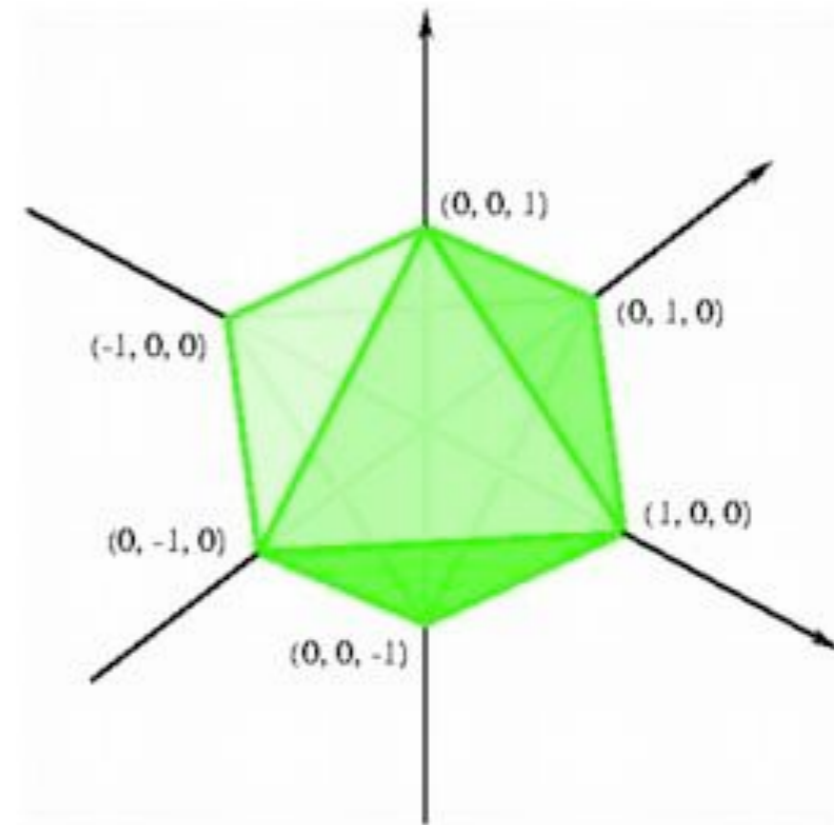
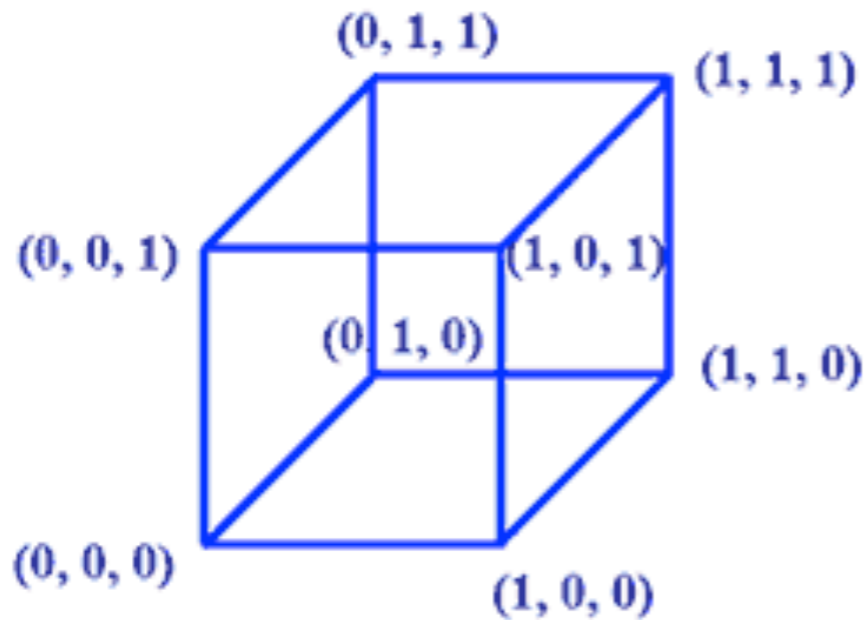
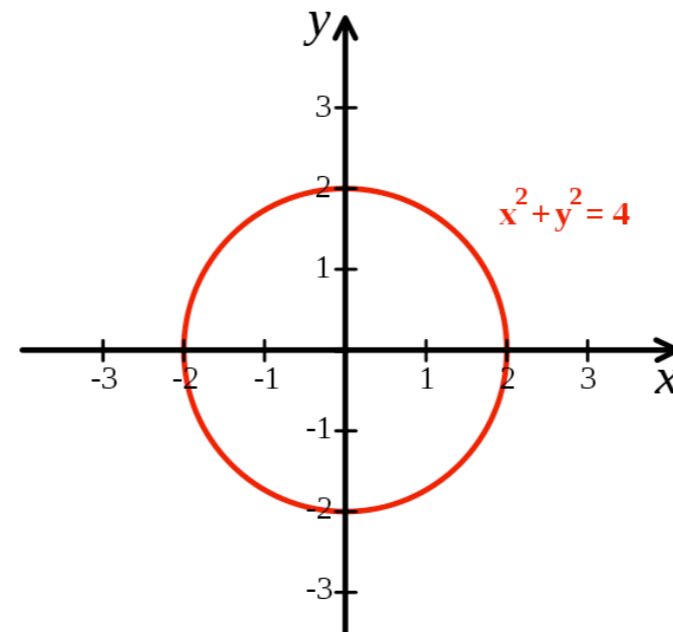
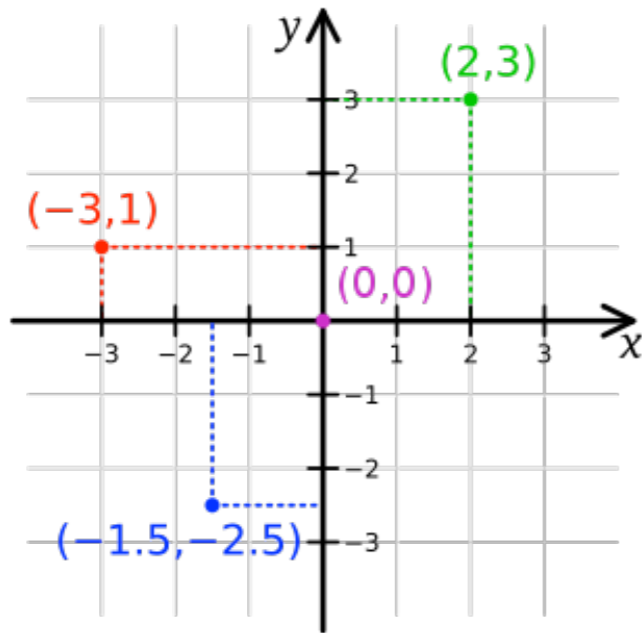
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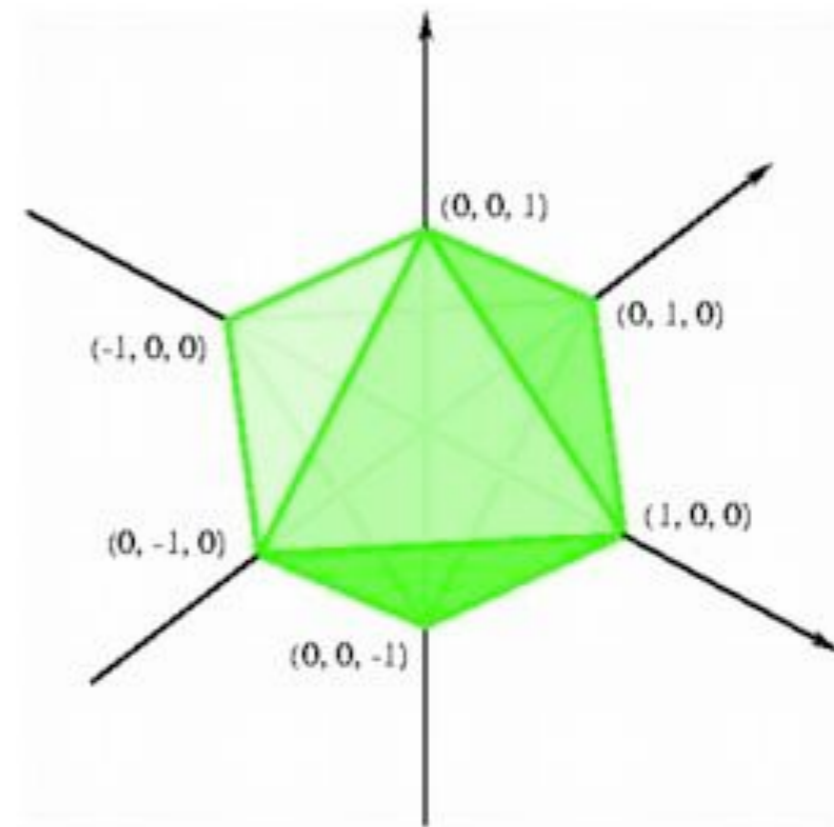
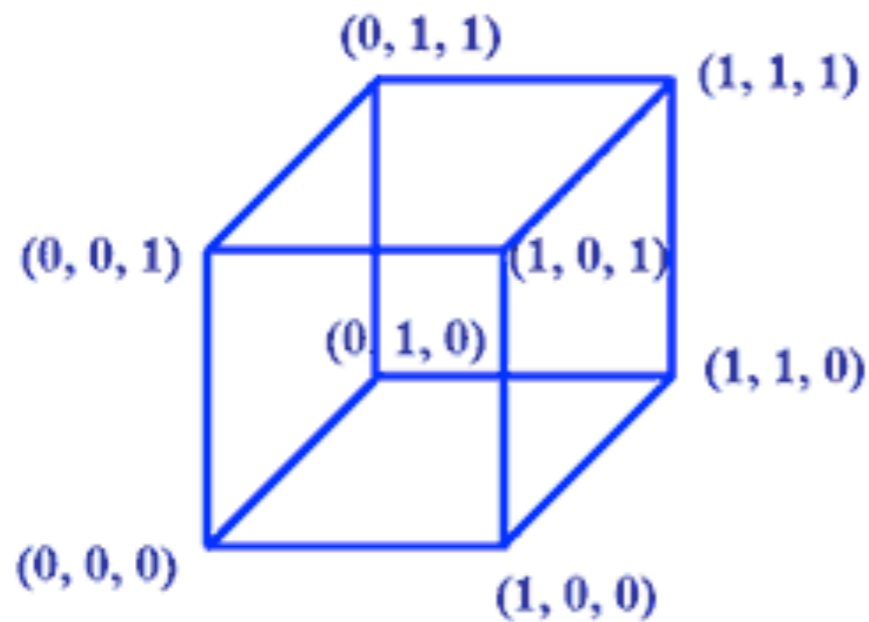
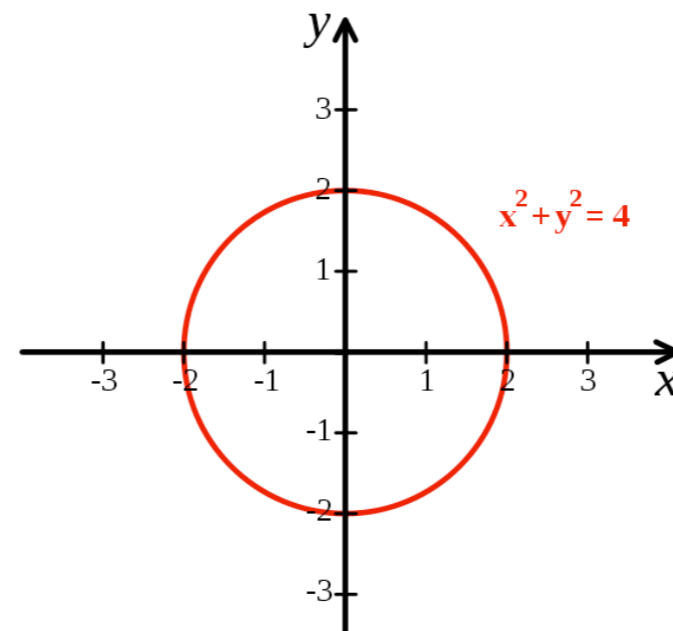
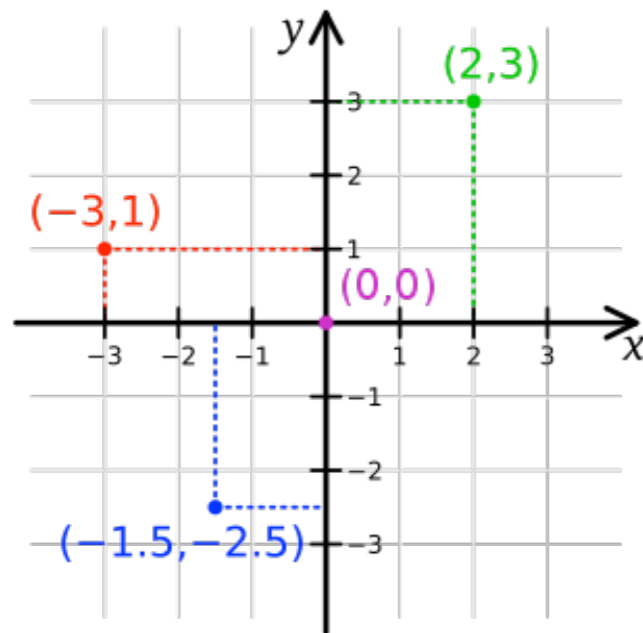
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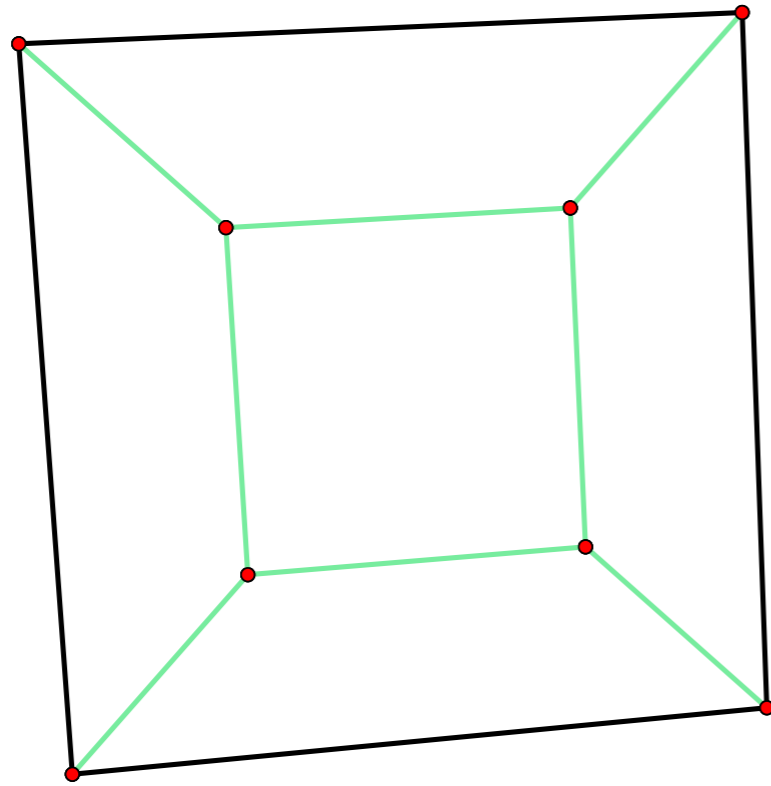
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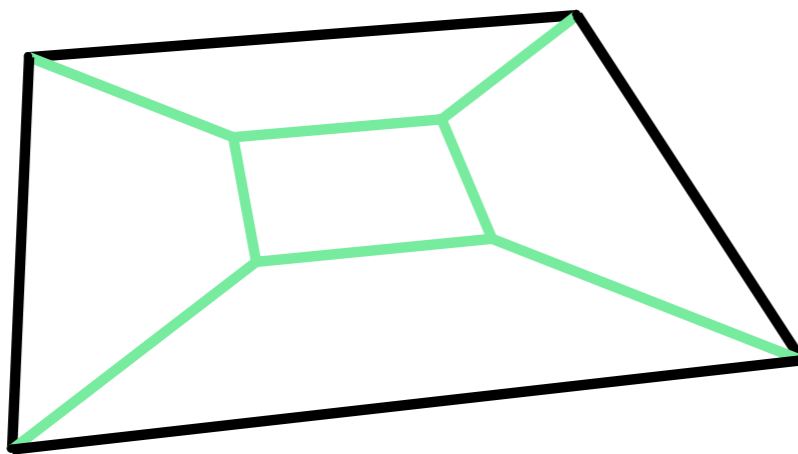
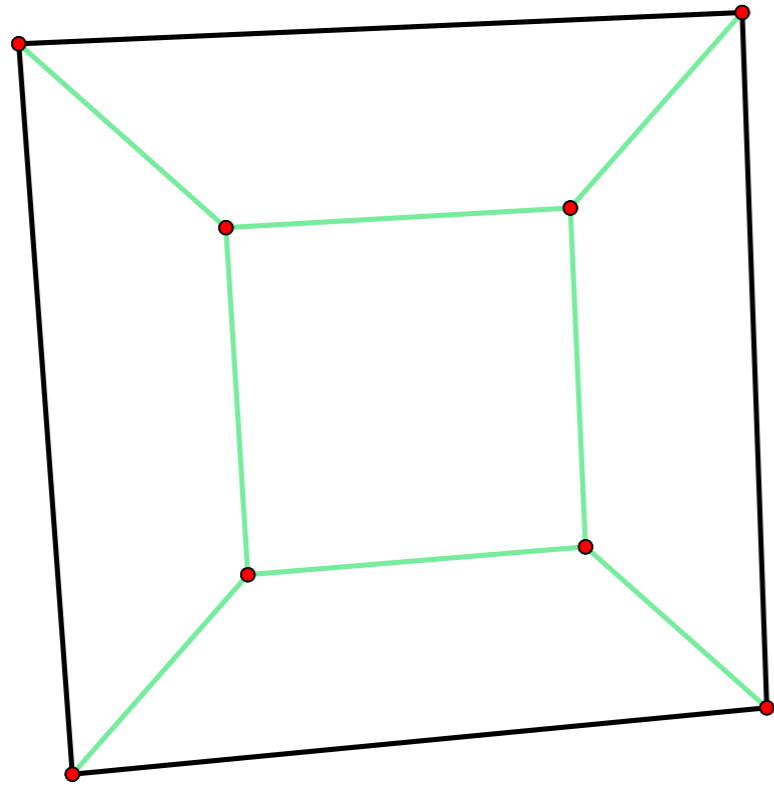
Polyeder = Lösungen von Systemen linearer Ungleichungen

# Schlegel-Projektion

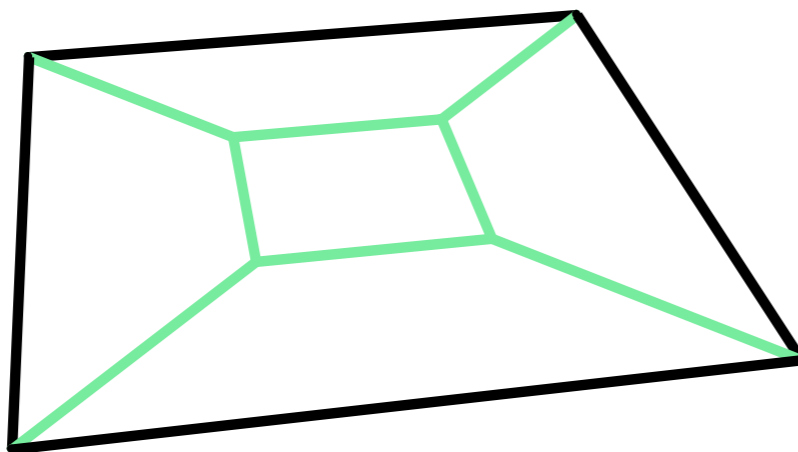
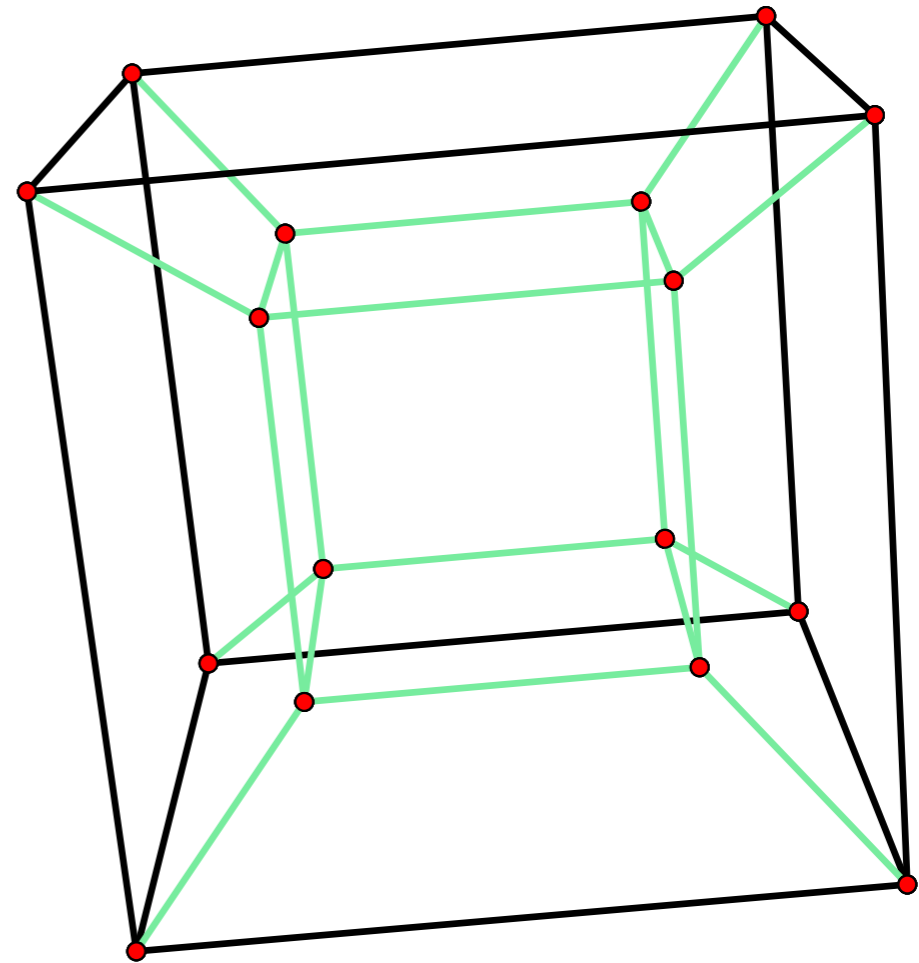
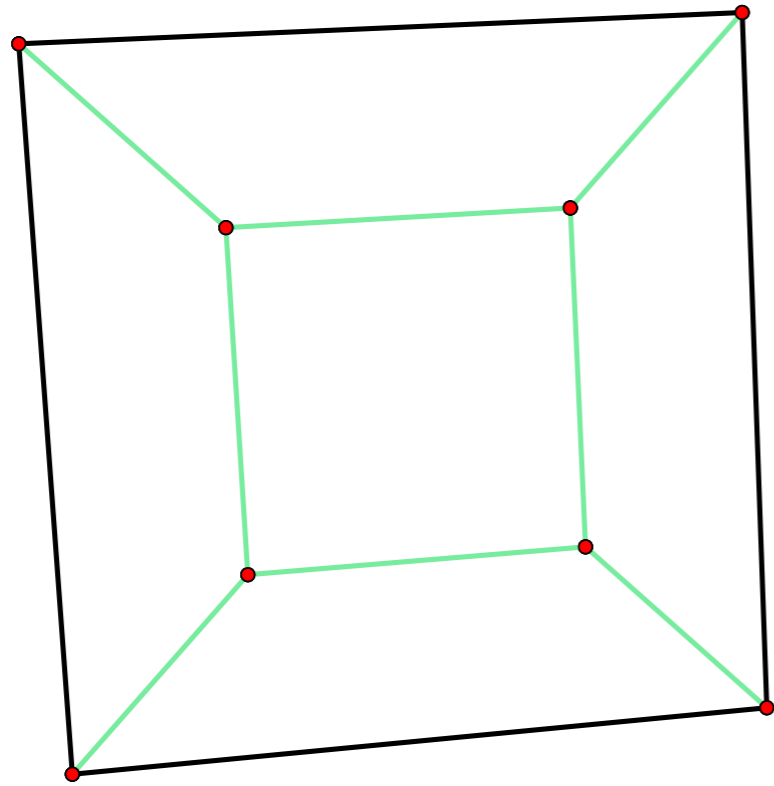
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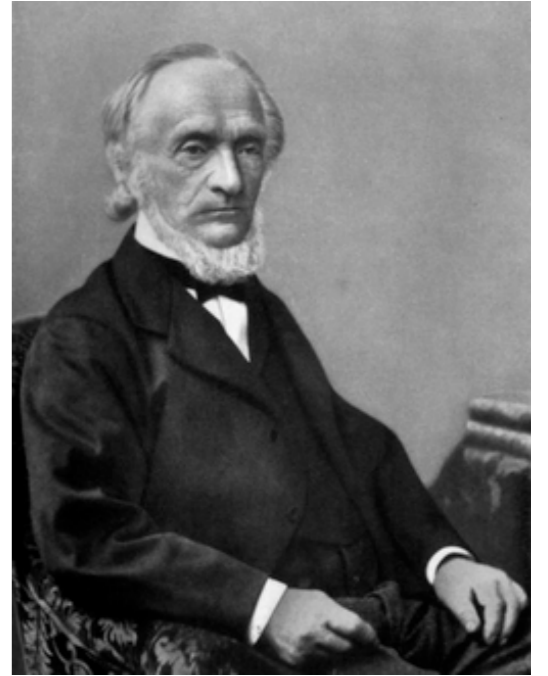


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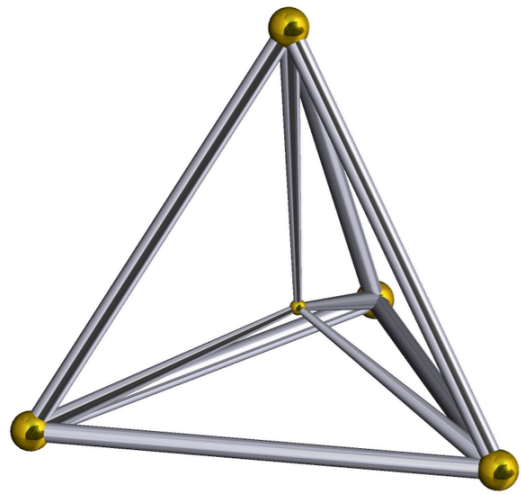


# Platonische Körper in 4D

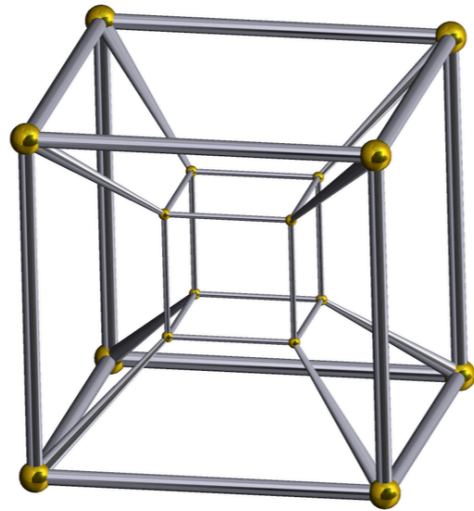


Ludwig Schläfli  
(1814-1895)

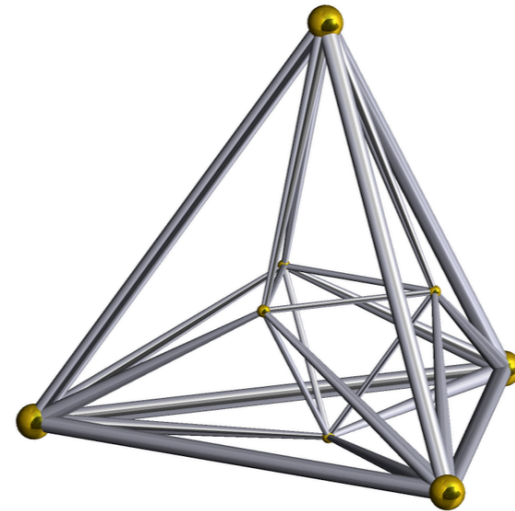
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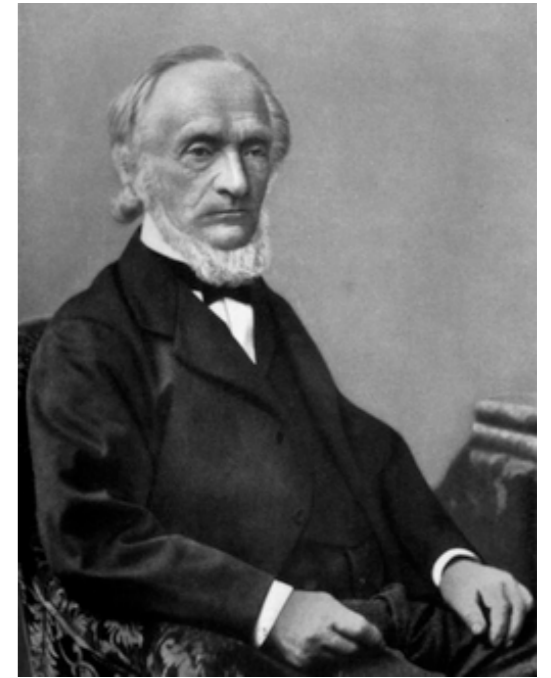
**5-Zell**



**8-Zell**

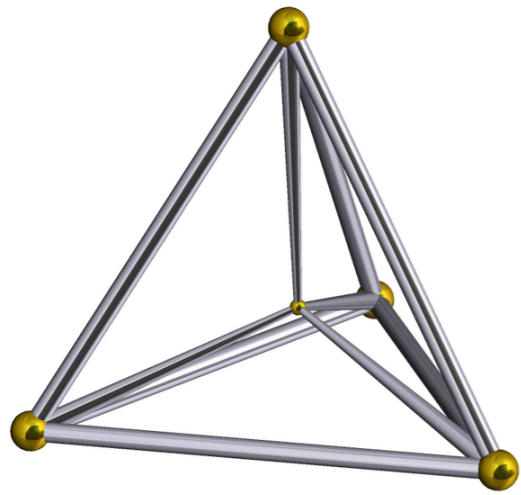


**16-Zell**

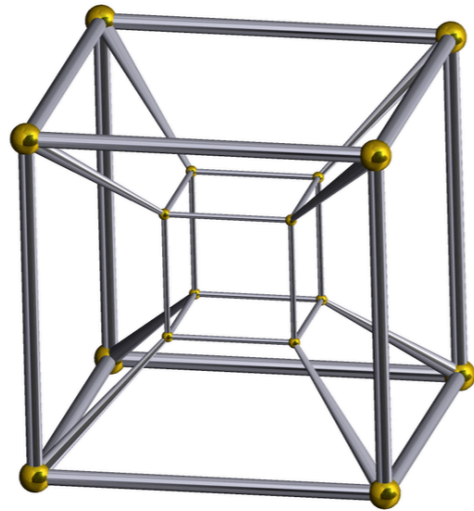


**Ludwig Schläfli  
(1814-1895)**

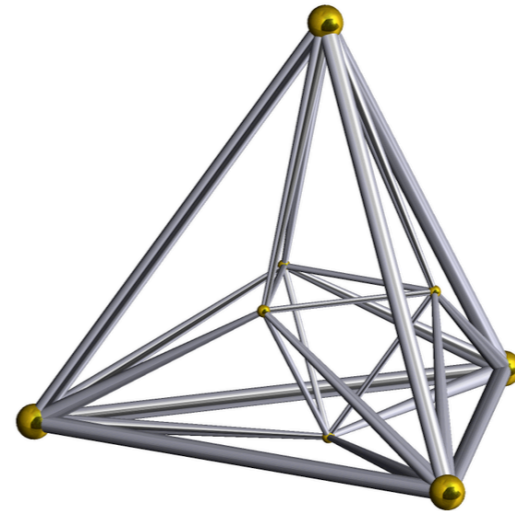
# Platonische Körper in 4D



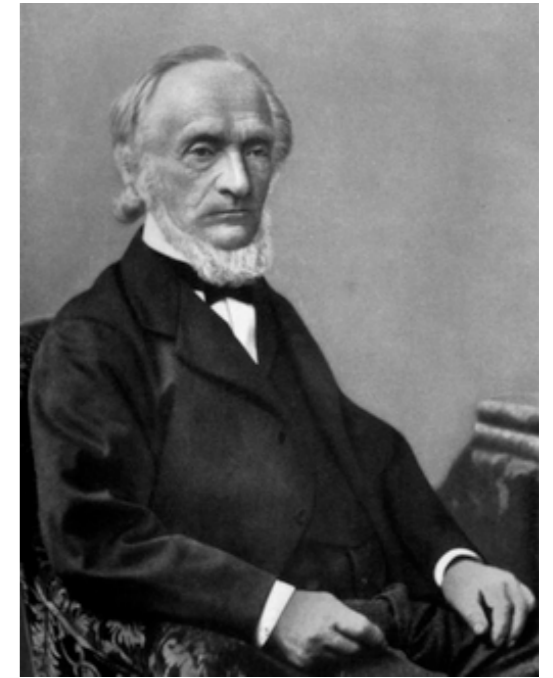
**5-Zell**



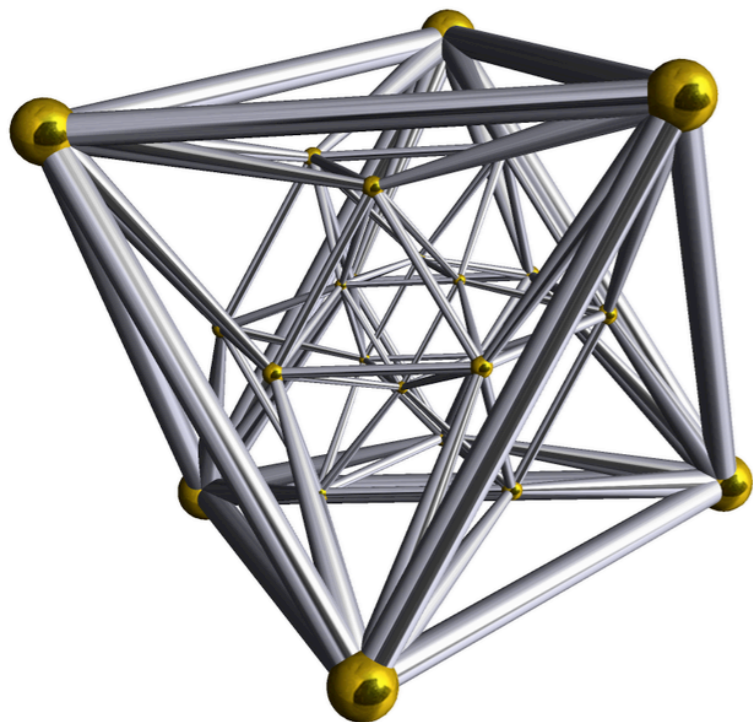
**8-Zell**



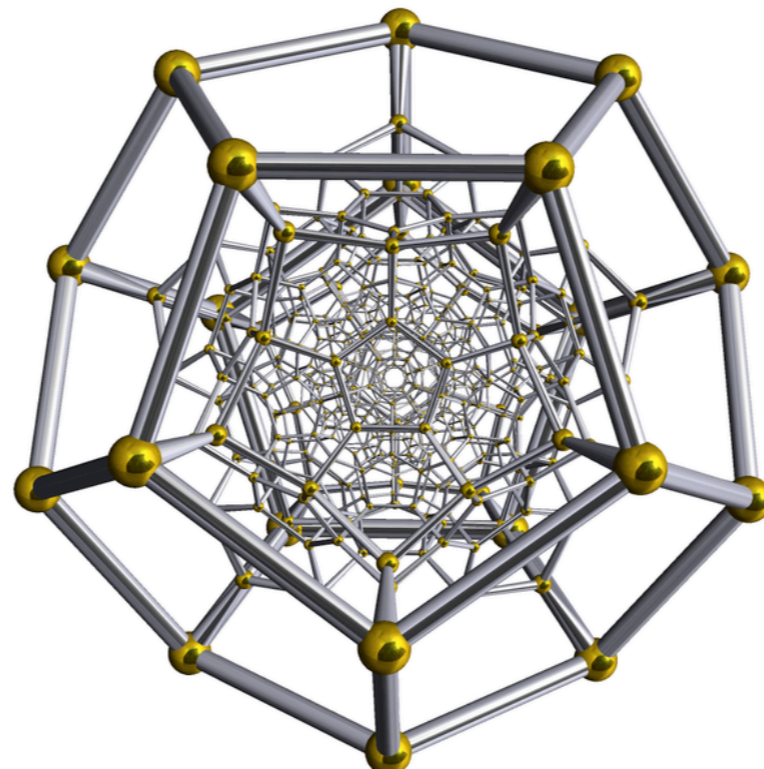
**16-Zell**



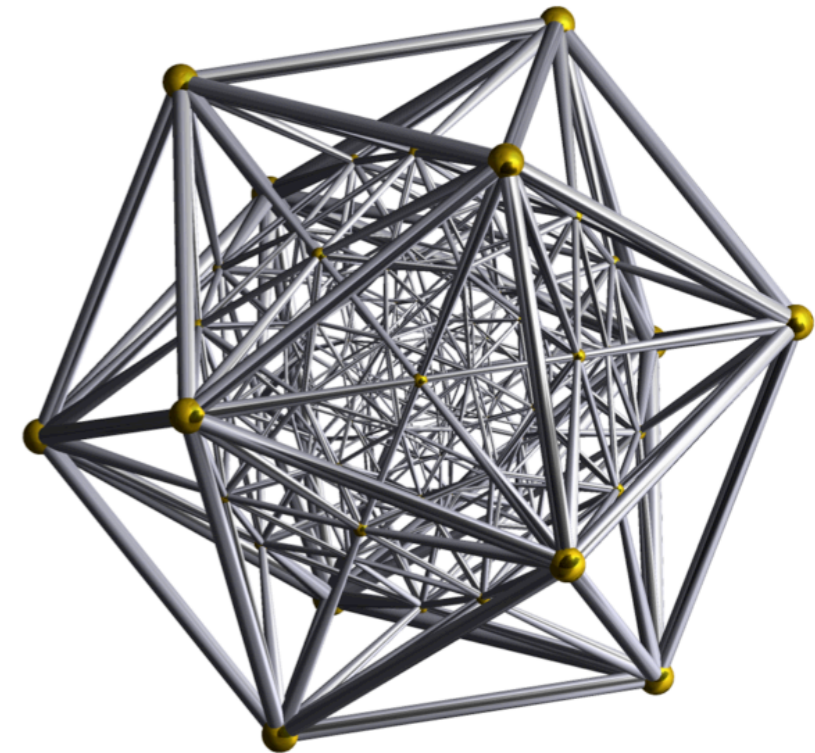
Ludwig Schläfli  
(1814-1895)



**24-Zell**



**120-Zell**



**600-Zell**

# Theorie in n Dimensionen

rechnen mit Vektoren  $(x_1, x_2, \dots, x_n)$

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rechnen mit Vektoren  $(x_1, x_2, \dots, x_n)$



Herrmann Minkowski  
(1864-1909)

Jedes **Polyeder** hat zwei Darstellungen:

- mit “Ecken”
- mit “Seitenflächen”

(linearen Ungleichungen)

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rechnen mit Vektoren  $(x_1, x_2, \dots, x_n)$

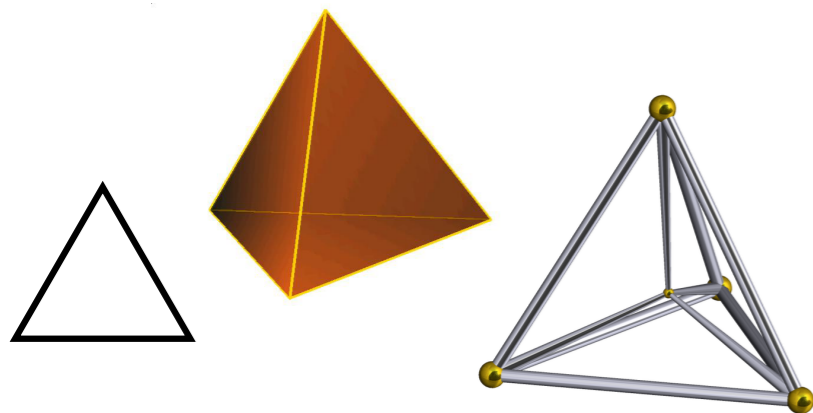


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**(n+1)-Zell**

Simplex

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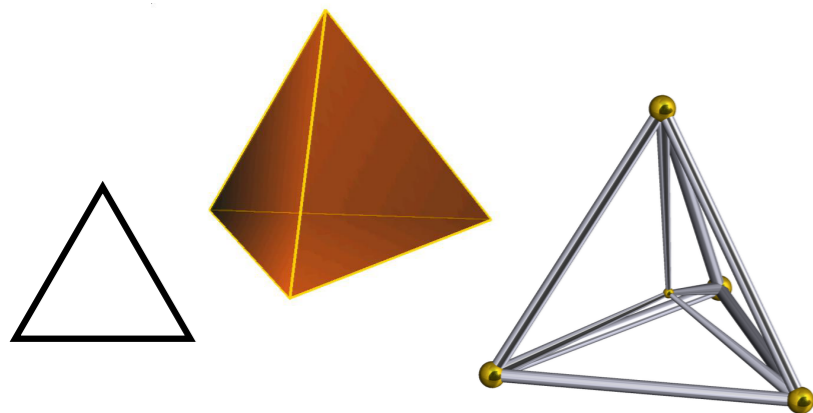


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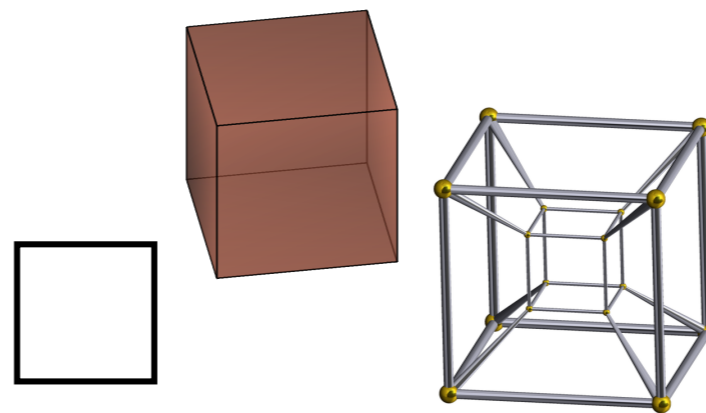
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**(n+1)-Zell**  
Simplex



**(2\*n)-Zell**  
(Hyper-)Würfel

# Theorie in n Dimensionen

rechnen mit Vektoren  $(x_1, x_2, \dots, x_n)$

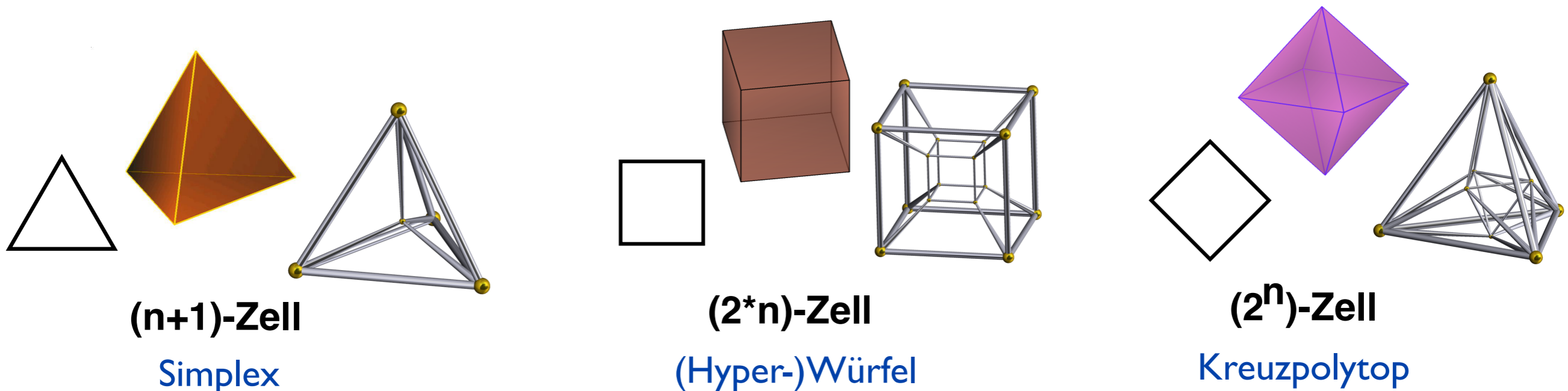


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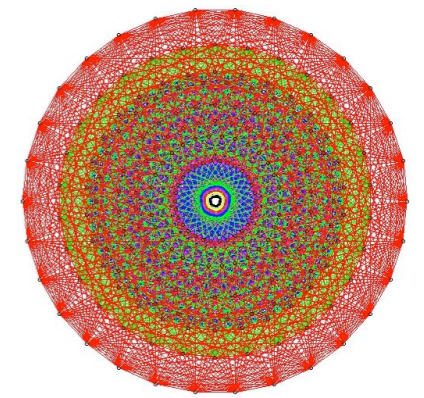
(linearen Ungleichungen)





# Theorie in n Dimensionen

rechnen mit Vektoren  $(x_1, x_2, \dots, x_n)$

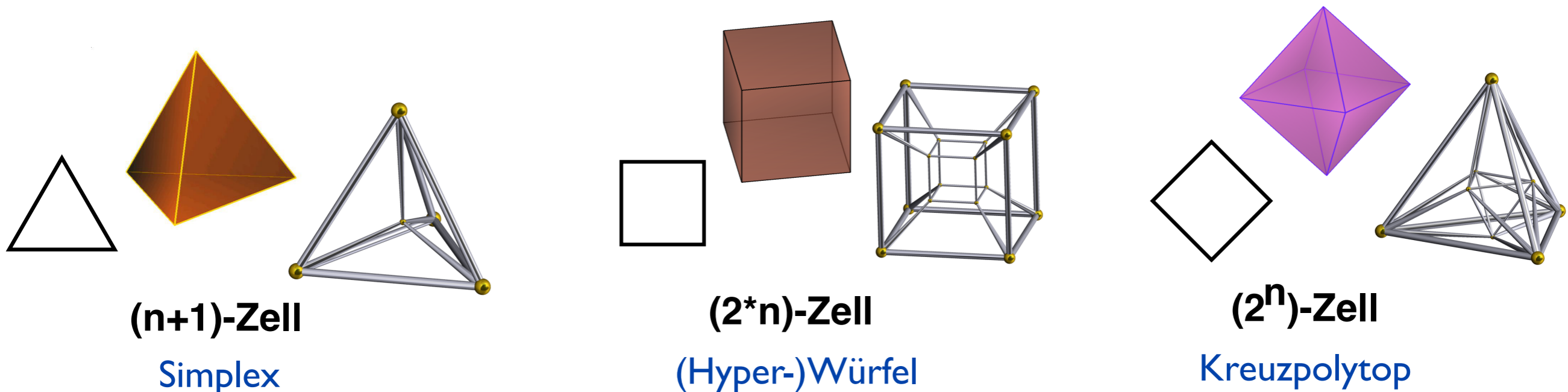


Herrmann Minkowski  
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Jedes **Polyeder** hat zwei Darstellungen:

- mit “Ecken”
- mit “Seitenflächen”

(linearen Ungleichungen)



**Was kann man damit machen?**

# Eine kleine Geschichte...

Was machen nach dem Abi?



# Geld verdienen mit den Omas?





# Geld verdienen mit den Omas?



			Max. Zeit	
<b>Oma 1</b> (stricken)	10	20	300	(Stunden)
<b>Oma 2</b> (nähen)	20	10	300	(Stunden)



# Geld verdienen mit den Omas?



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**Aufgabe:** Maximiere  $30x + 40y$

# Geld verdienen mit den Omas?



			Max. Zeit	
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<b>Preise</b>	30	40		(Euro)

## Aufgabe:

Maximiere  $30x + 40y$



wobei  $10x + 20y \leq 300$  (Oma 1)

$20x + 10y \leq 300$  (Oma 2)



# Geld verdienen mit den Omas?



			Max. Zeit	
<b>Oma 1</b> (stricken)	10	20	300	(Stunden)
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<b>Preise</b>	30	40		(Euro)

Maximiere  $3x + 4y$

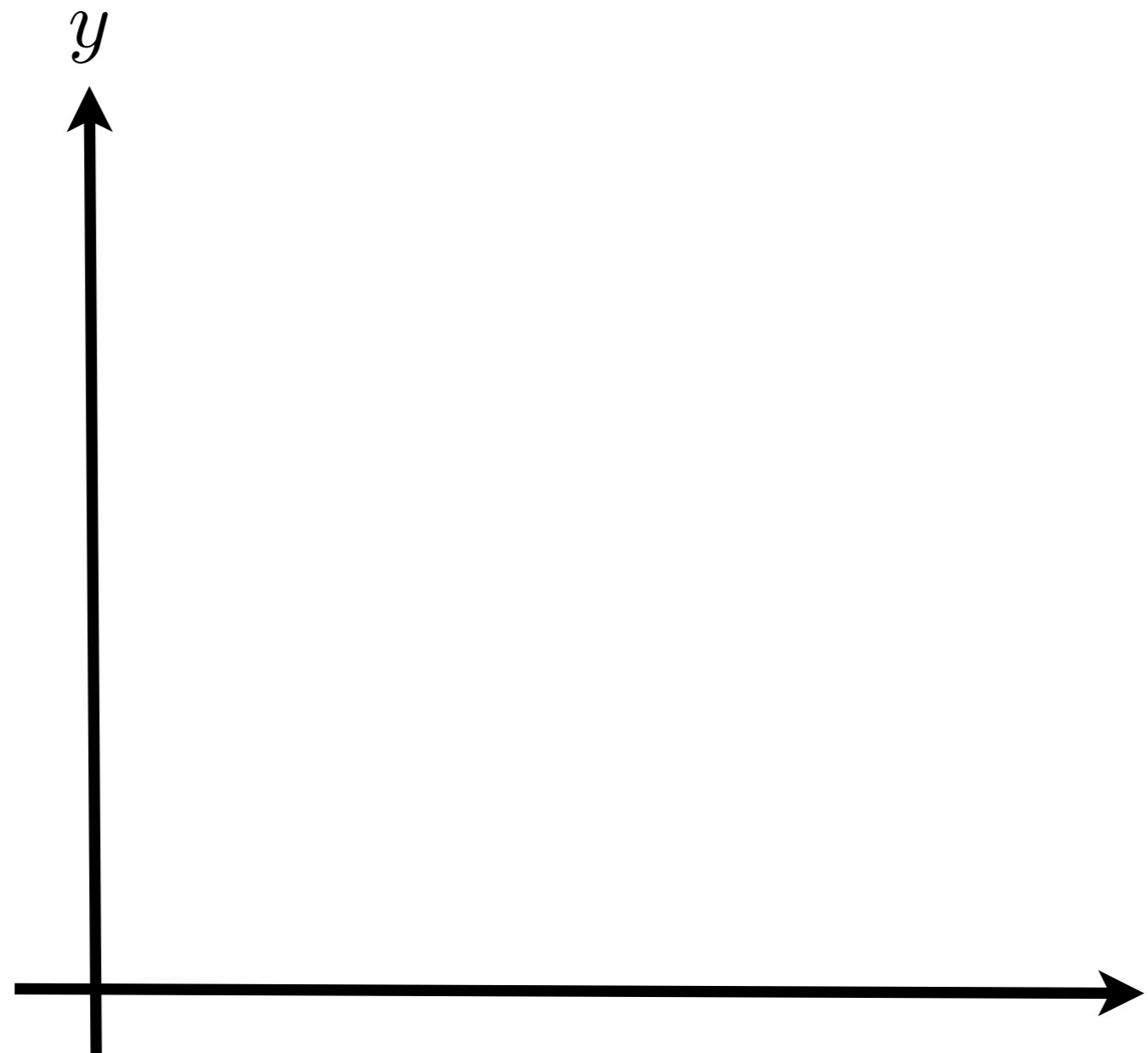
$$x + 2y \leq 30$$

$$2x + y \leq 30$$

# Geometrie hilft...

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Anzahl Pullover 2



$x$

Anzahl  
Pullover 1

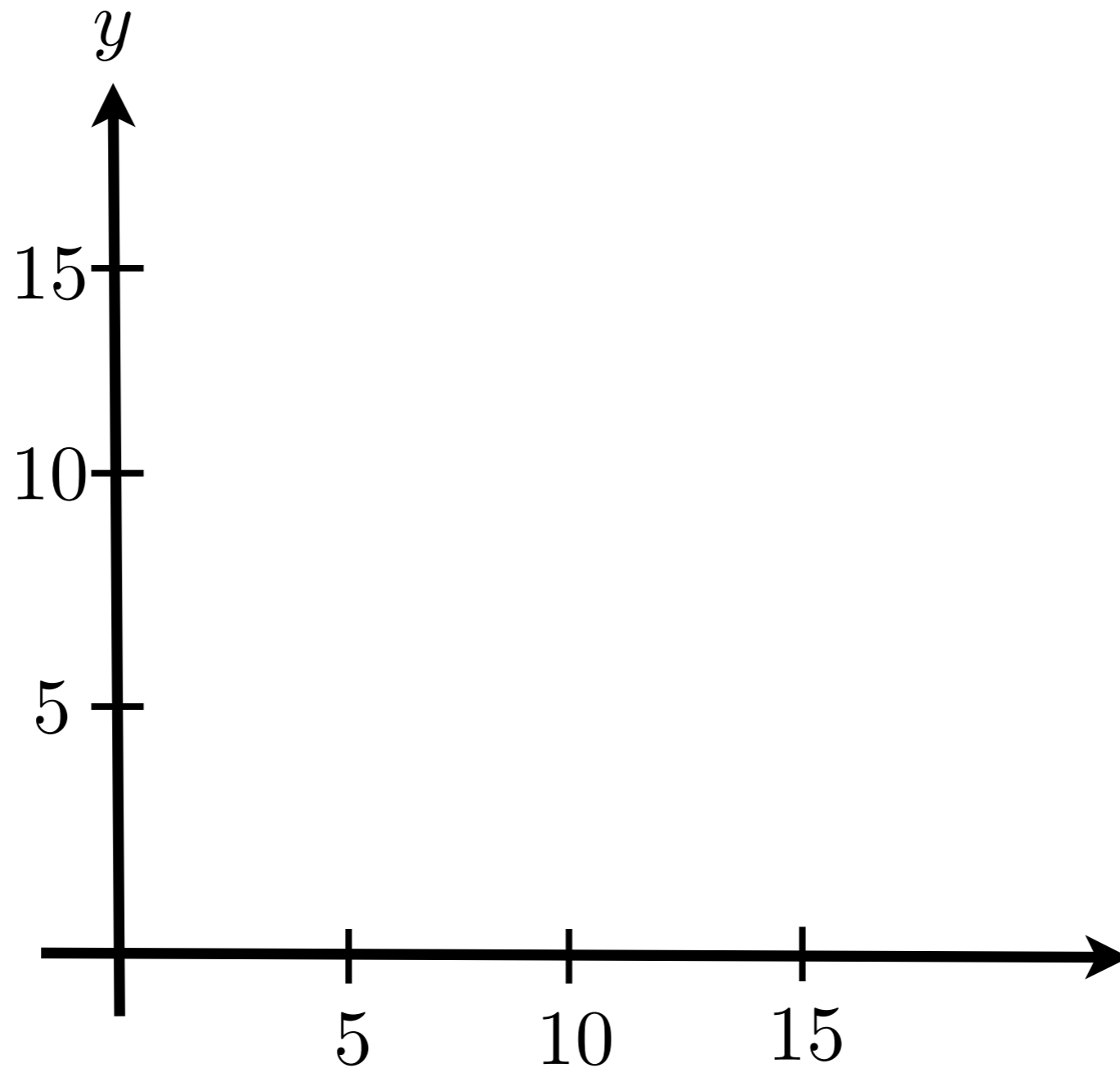
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Maximiere  $3x + 4y$

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Anzahl Pullover 2



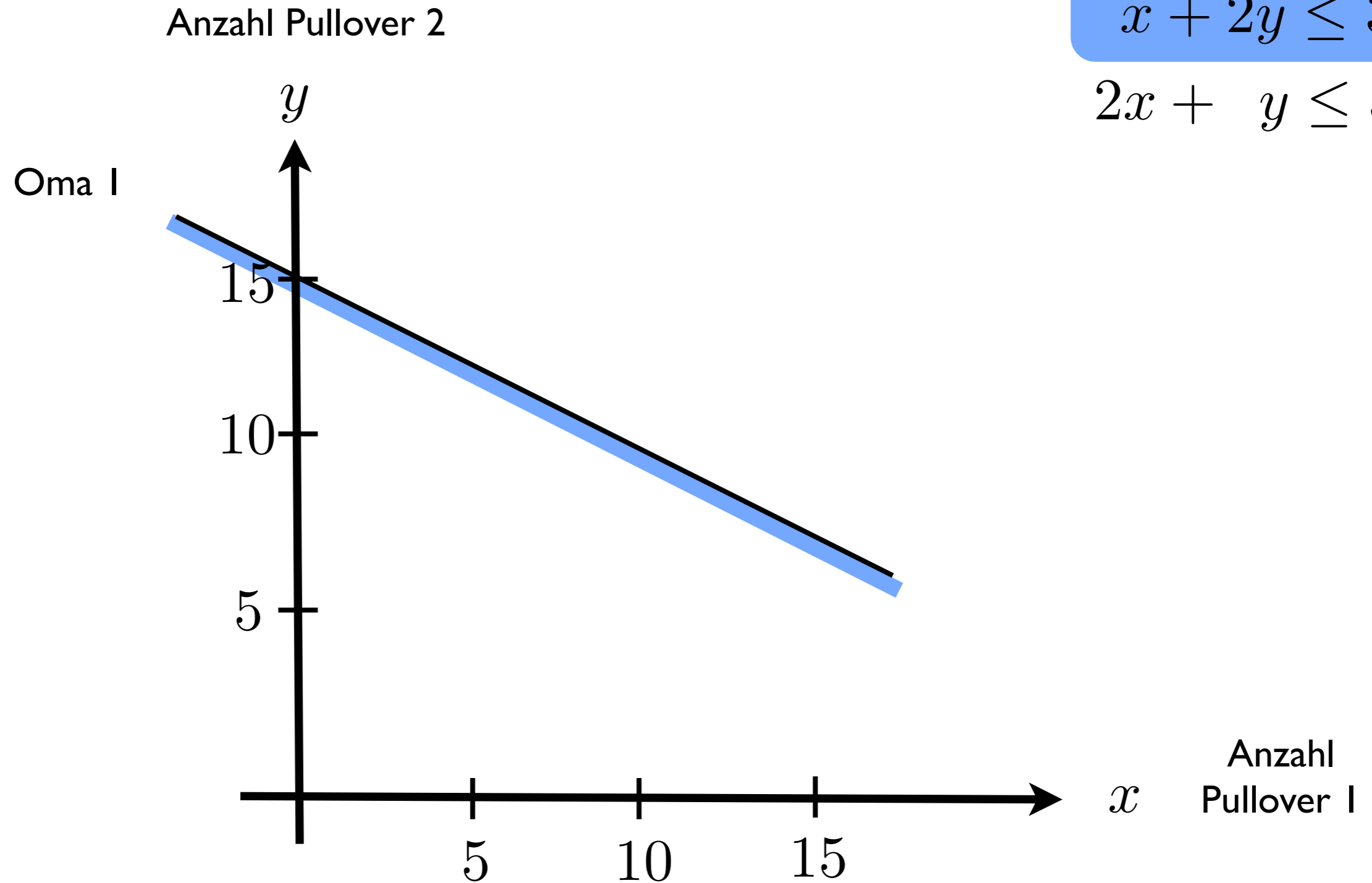
Anzahl  
Pullover 1

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$$x + 2y \leq 30$$

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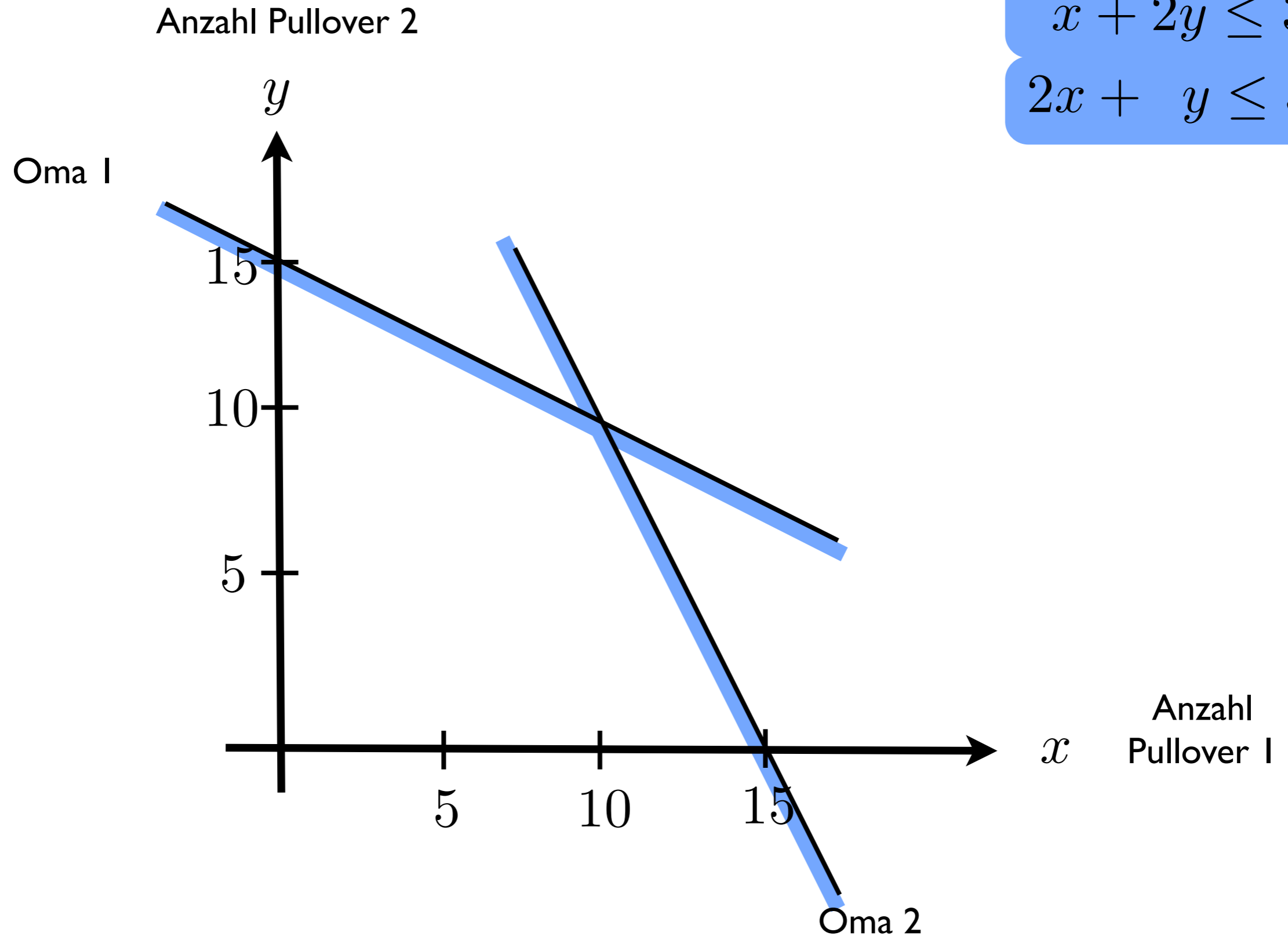


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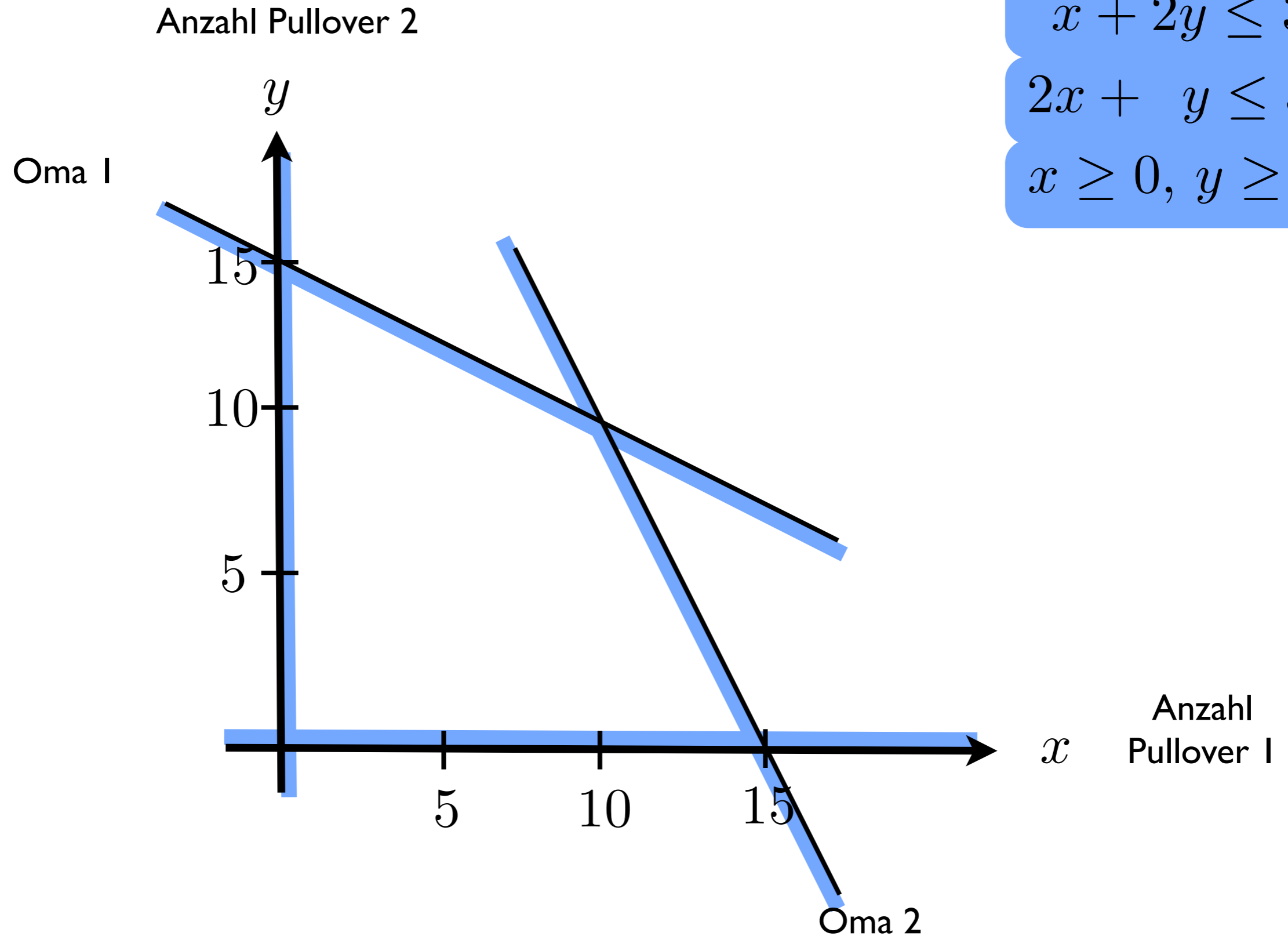
# Geometrie hilft...

Maximiere  $3x + 4y$

$$x + 2y \leq 30$$

$$2x + y \leq 30$$

$$x \geq 0, y \geq 0$$



# Geometrie hilft...

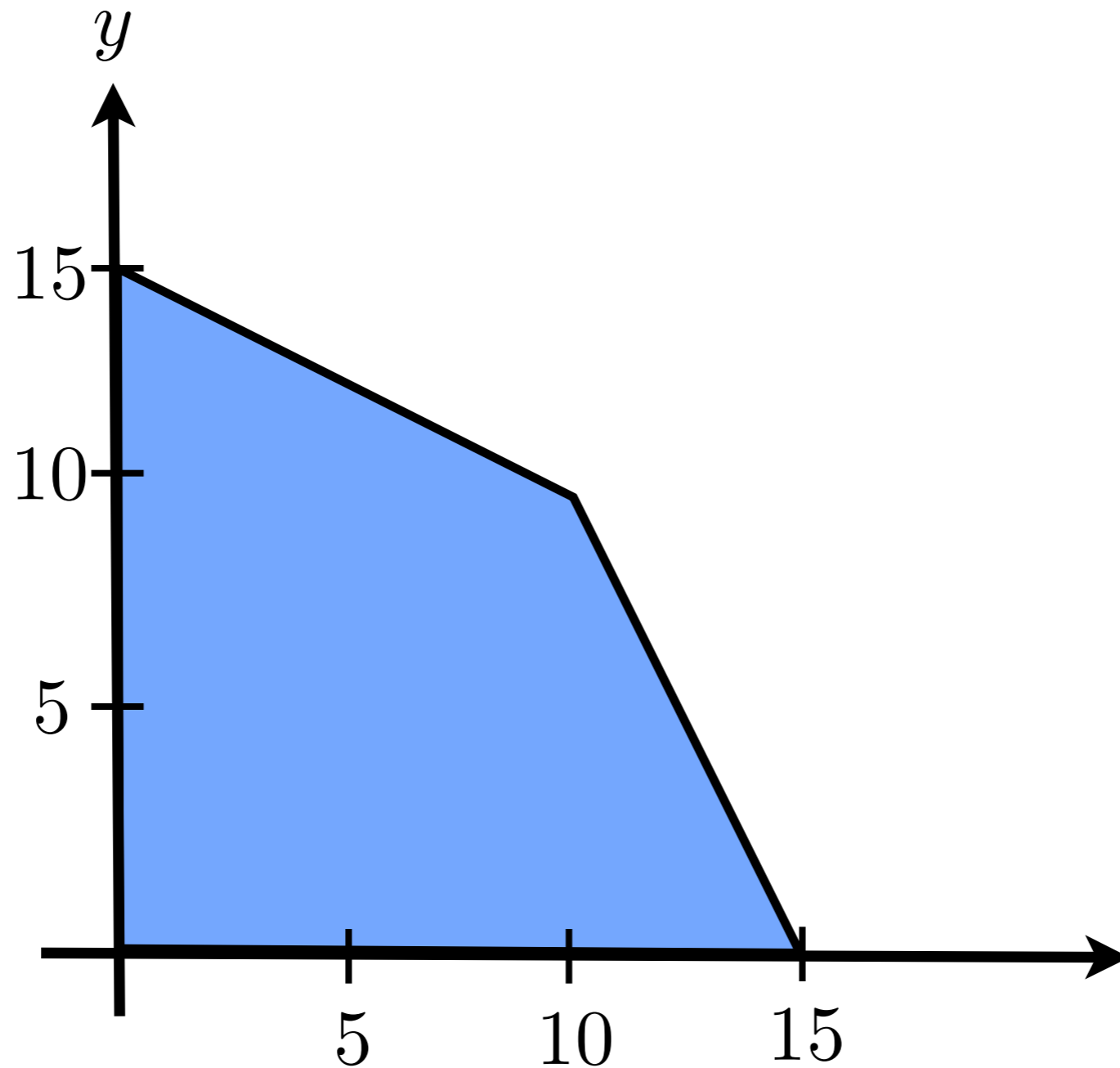
Maximiere  $3x + 4y$

$$x + 2y \leq 30$$

$$2x + y \leq 30$$

$$x \geq 0, y \geq 0$$

Anzahl Pullover 2



$x$

Anzahl  
Pullover 1



# Geometrie hilft...

Maximiere

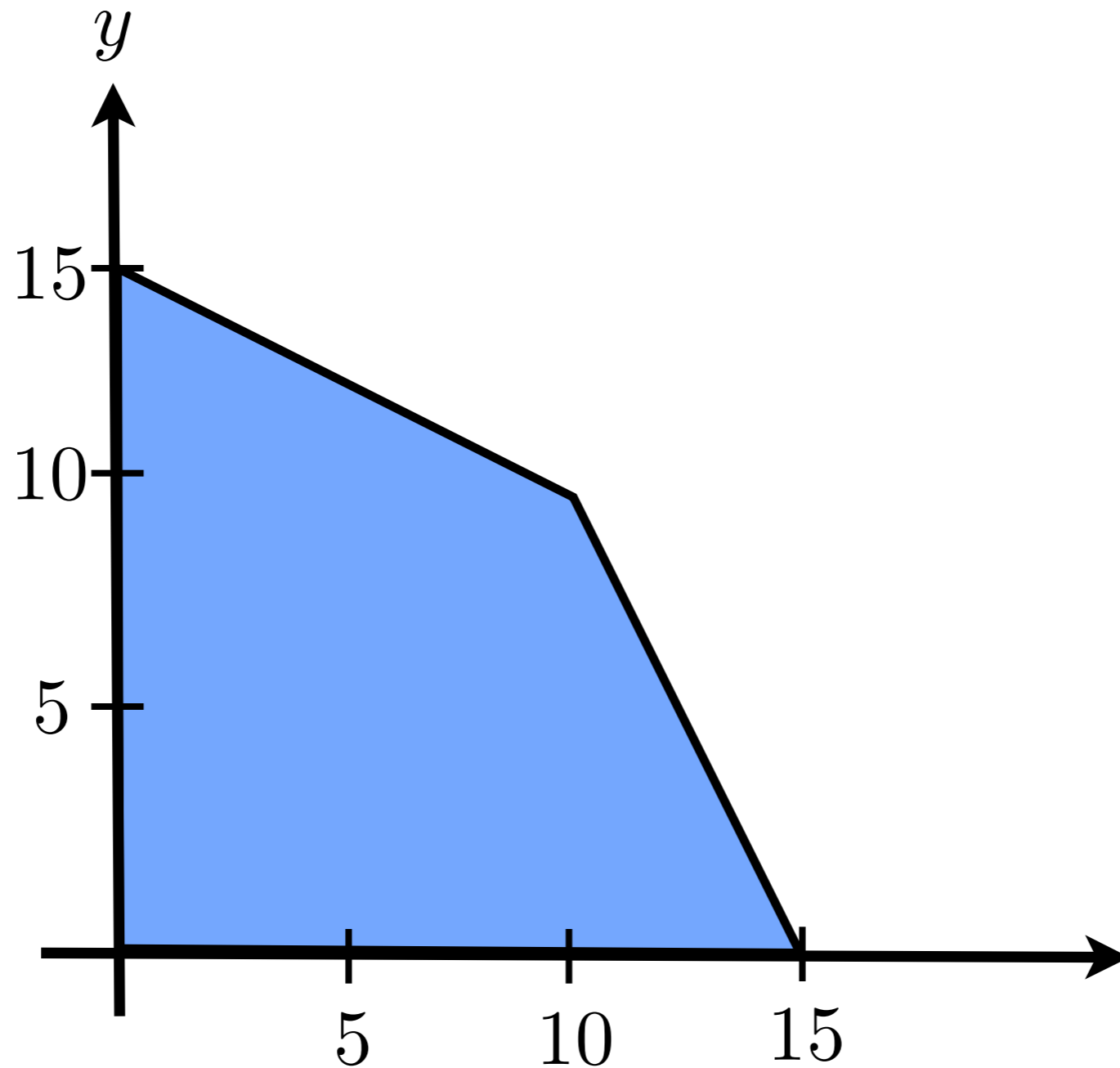
$$3x + 4y$$

$$x + 2y \leq 30$$

$$2x + y \leq 30$$

$$x \geq 0, y \geq 0$$

Anzahl Pullover 2



Anzahl  
Pullover 1

# Geometrie hilft...

Maximiere

$$3x + 4y$$

$$x + 2y \leq 30$$

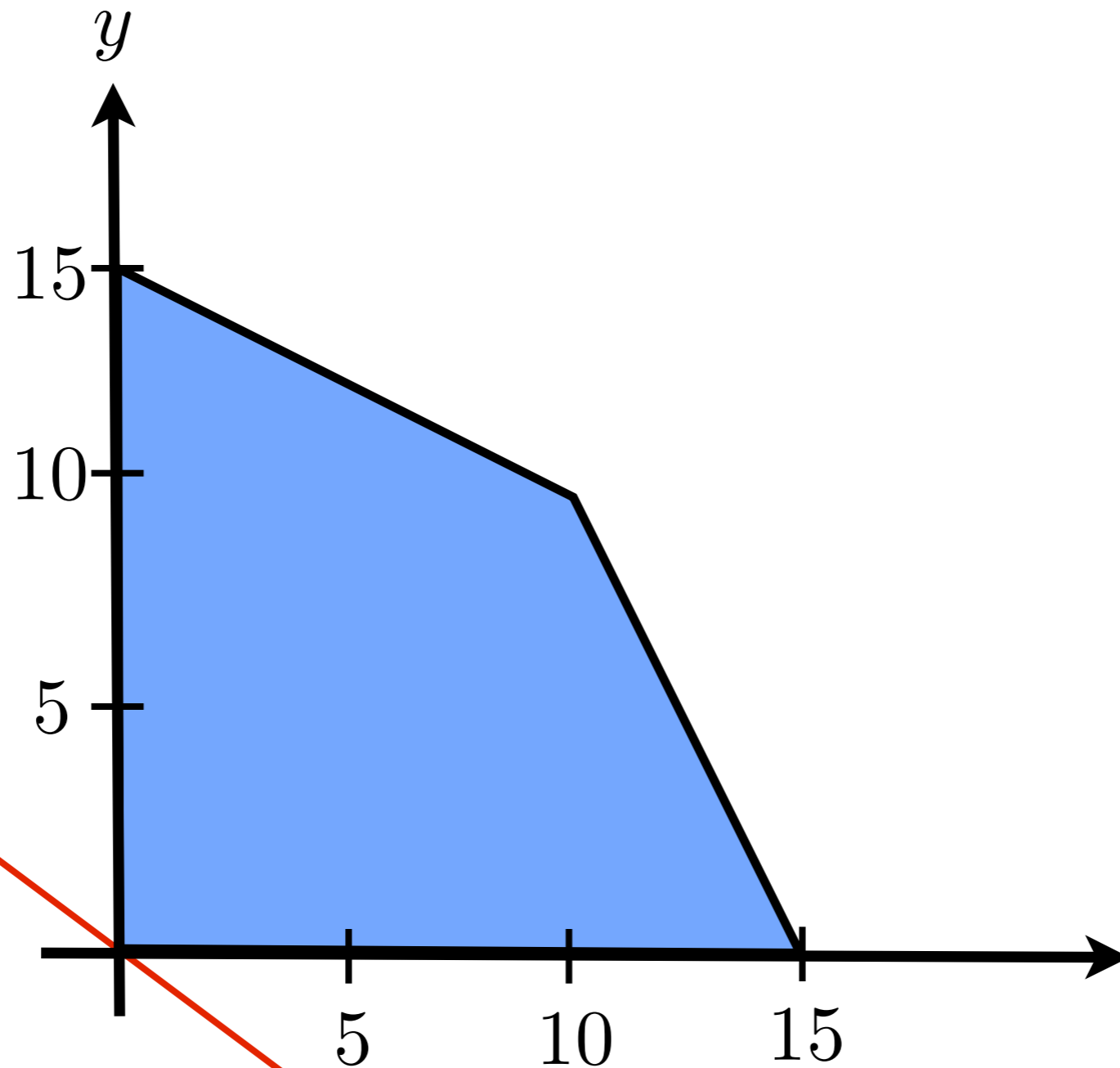
$$2x + y \leq 30$$

$$x \geq 0, y \geq 0$$

Anzahl Pullover 2

$$3x + 4y =$$

0



Anzahl  
Pullover 1

# Geometrie hilft...

Maximiere

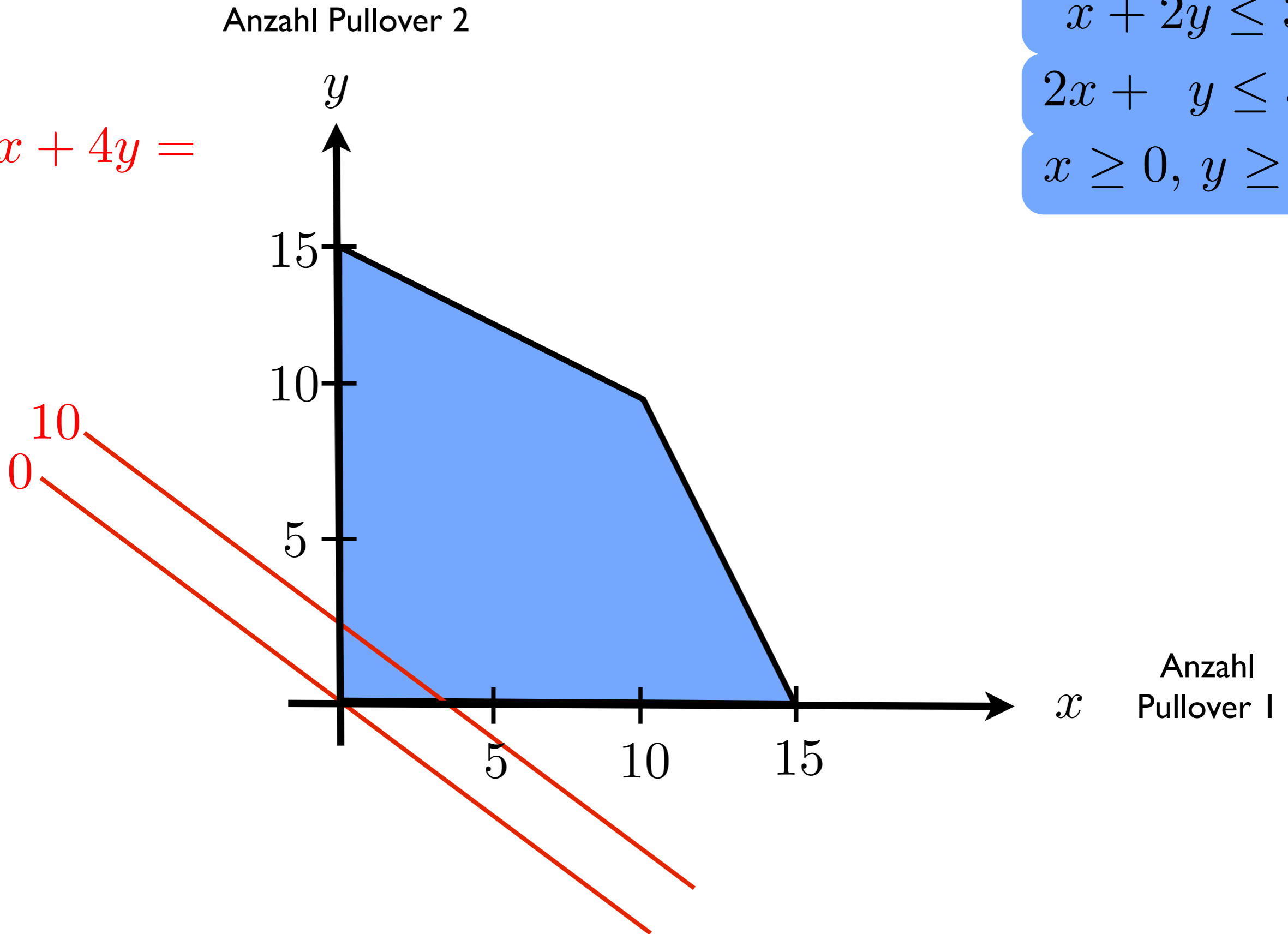
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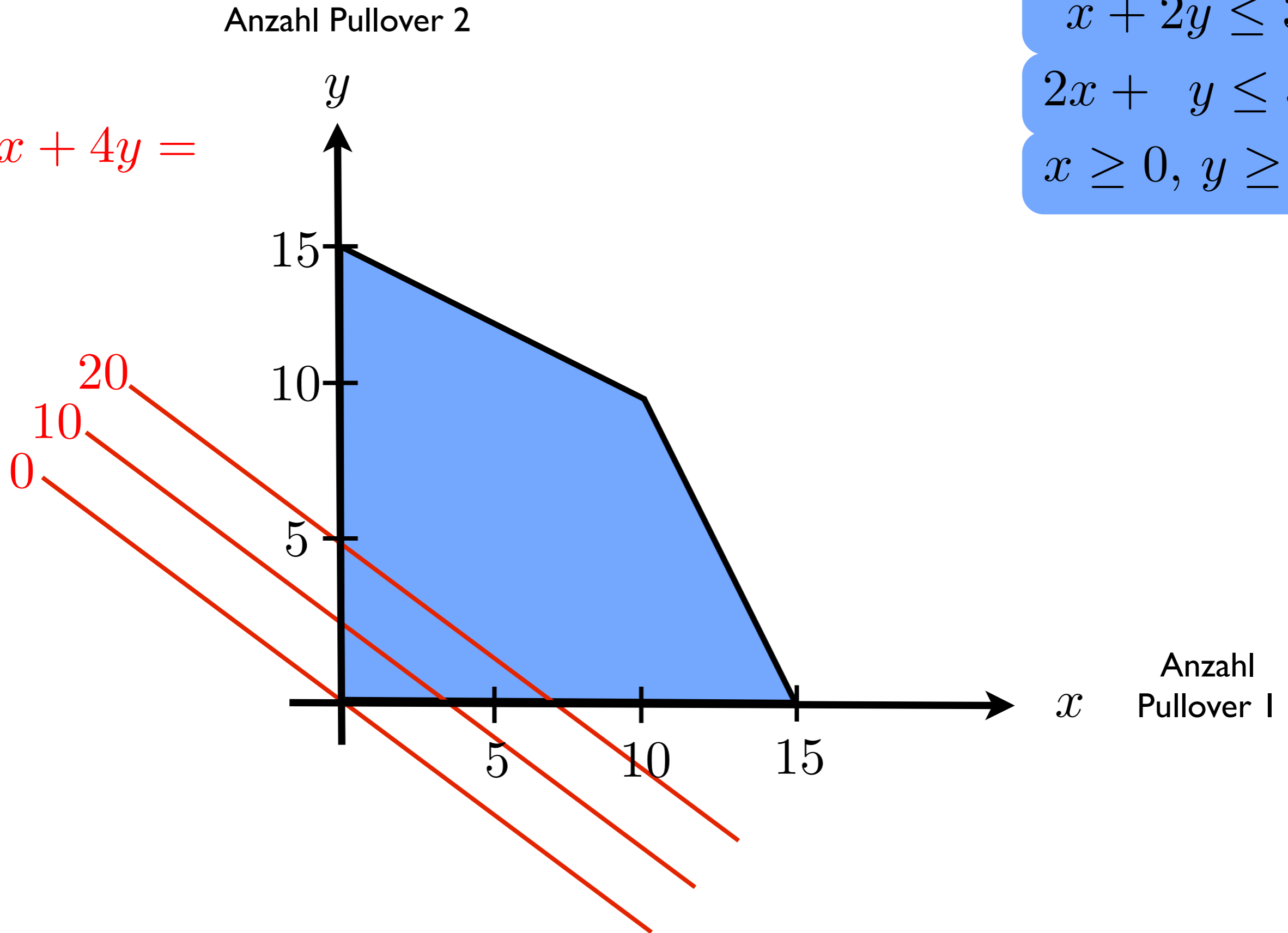
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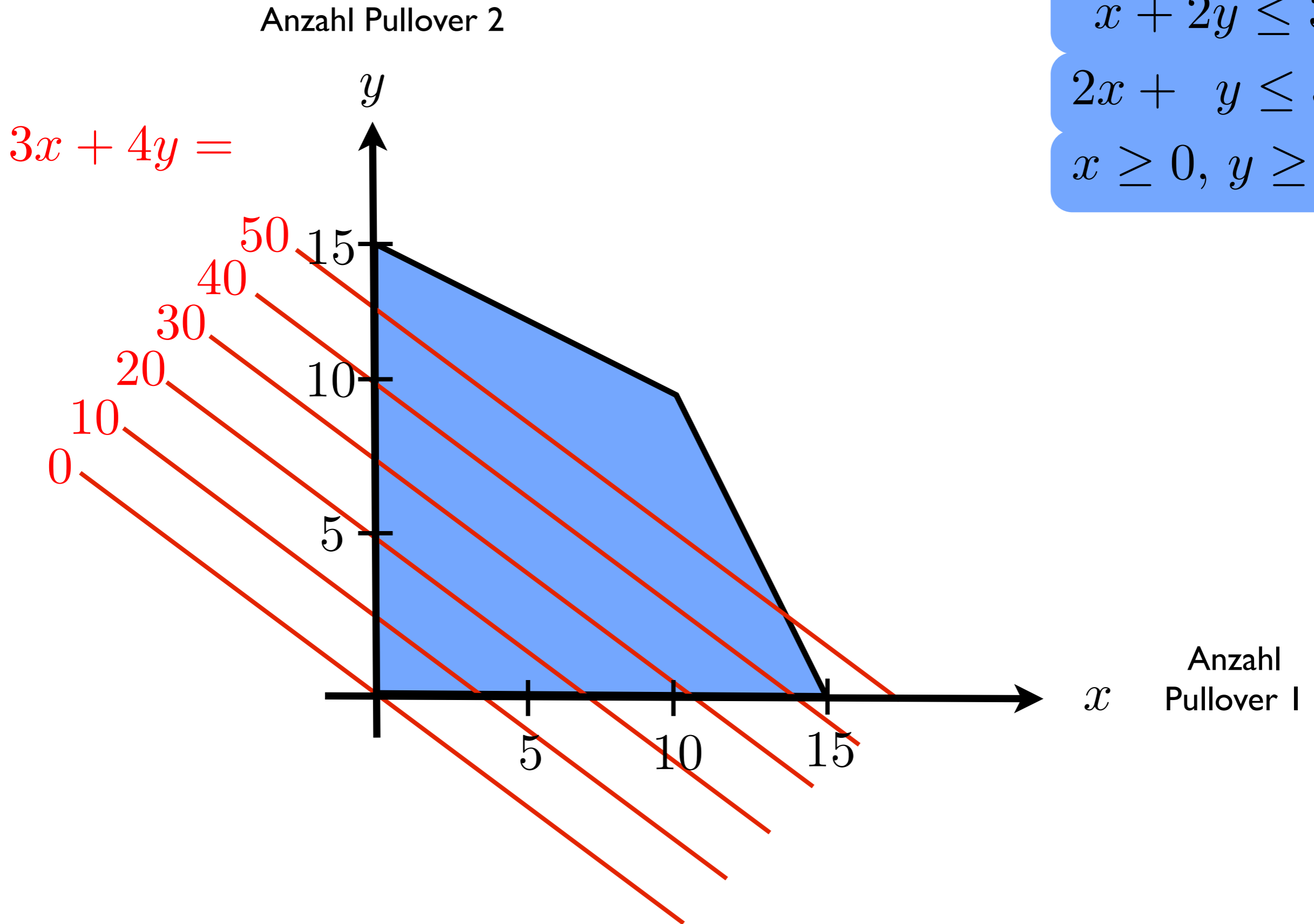
Maximiere

$$3x + 4y$$

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$$2x + y \leq 30$$

$$x \geq 0, y \geq 0$$



# Geometrie hilft...

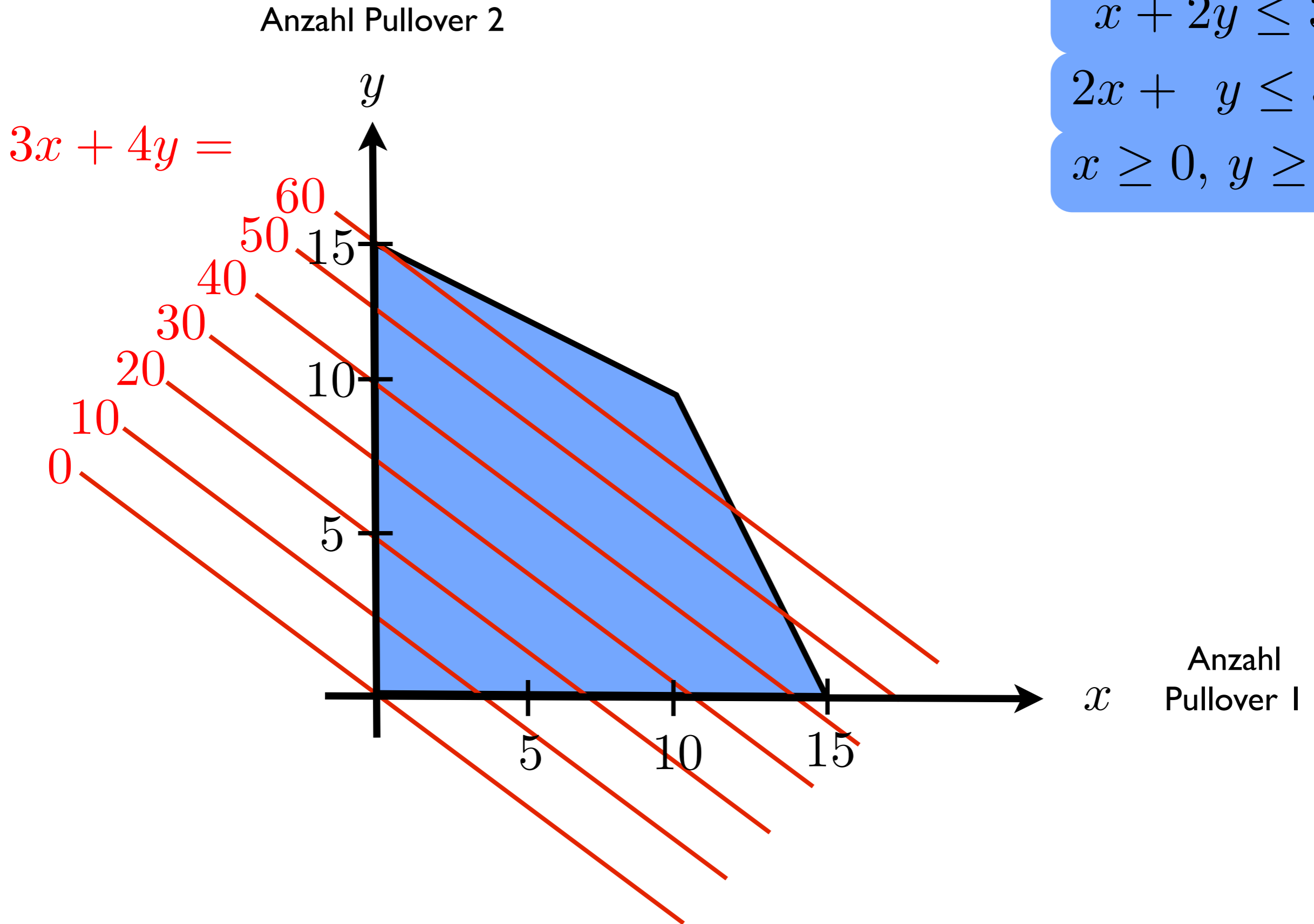
Maximiere

$$3x + 4y$$

$$x + 2y \leq 30$$

$$2x + y \leq 30$$

$$x \geq 0, y \geq 0$$



# Geometrie hilft...

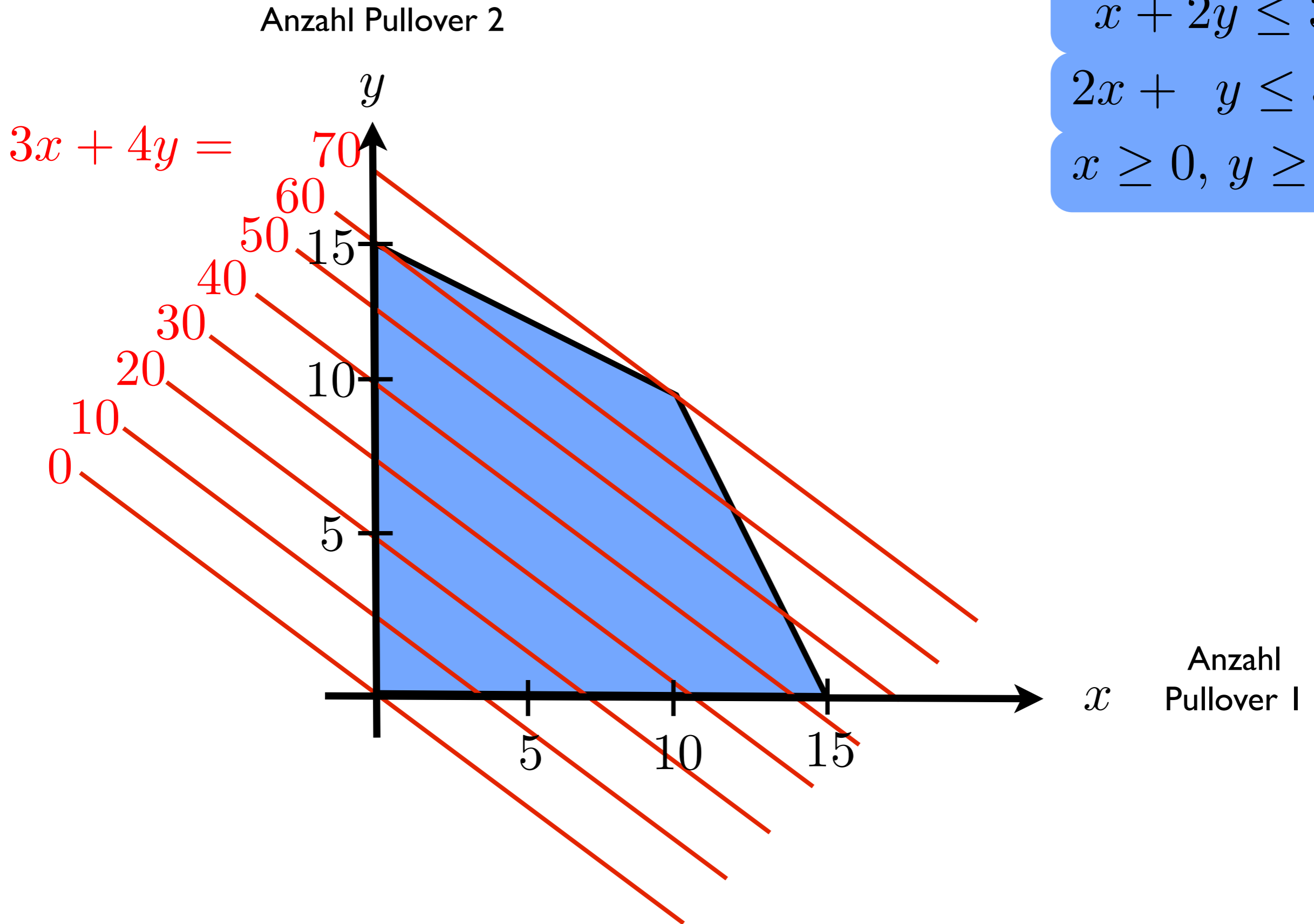
Maximiere

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# Geometrie hilft...

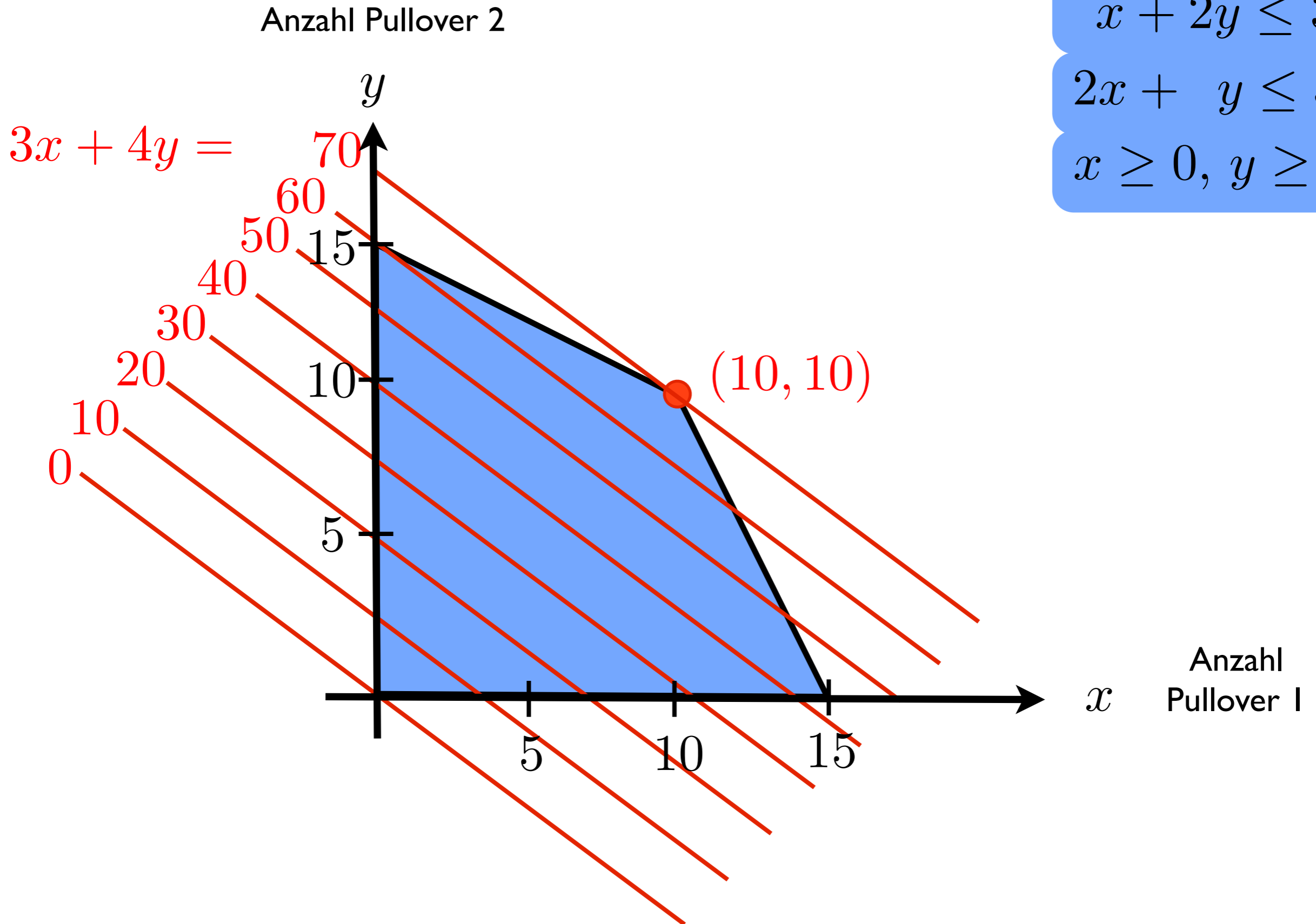
Maximiere

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# Geometrie hilft...

Maximiere

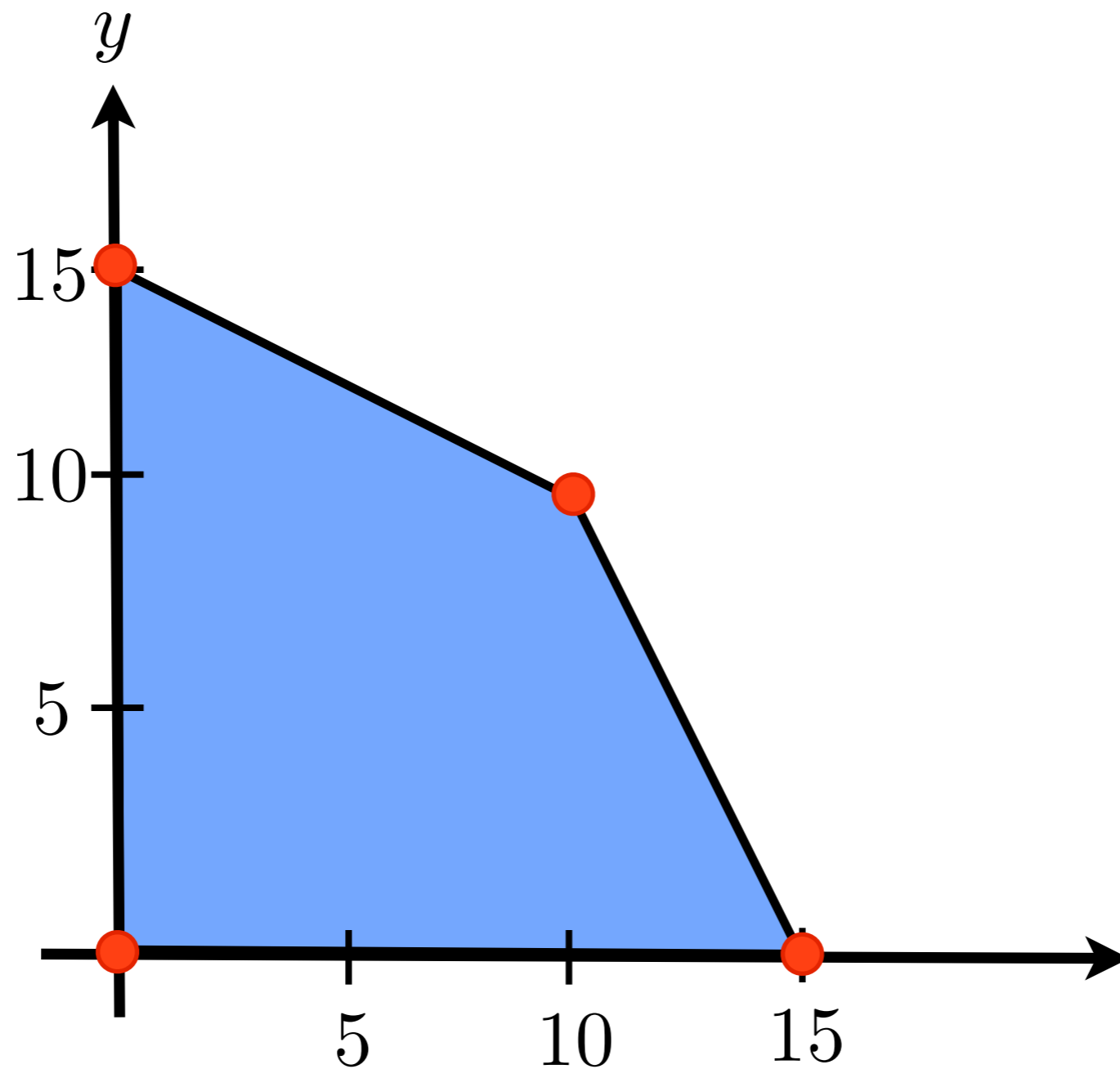


$$x + 2y \leq 30$$

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Anzahl Pullover 2



$x$

Anzahl  
Pullover 1

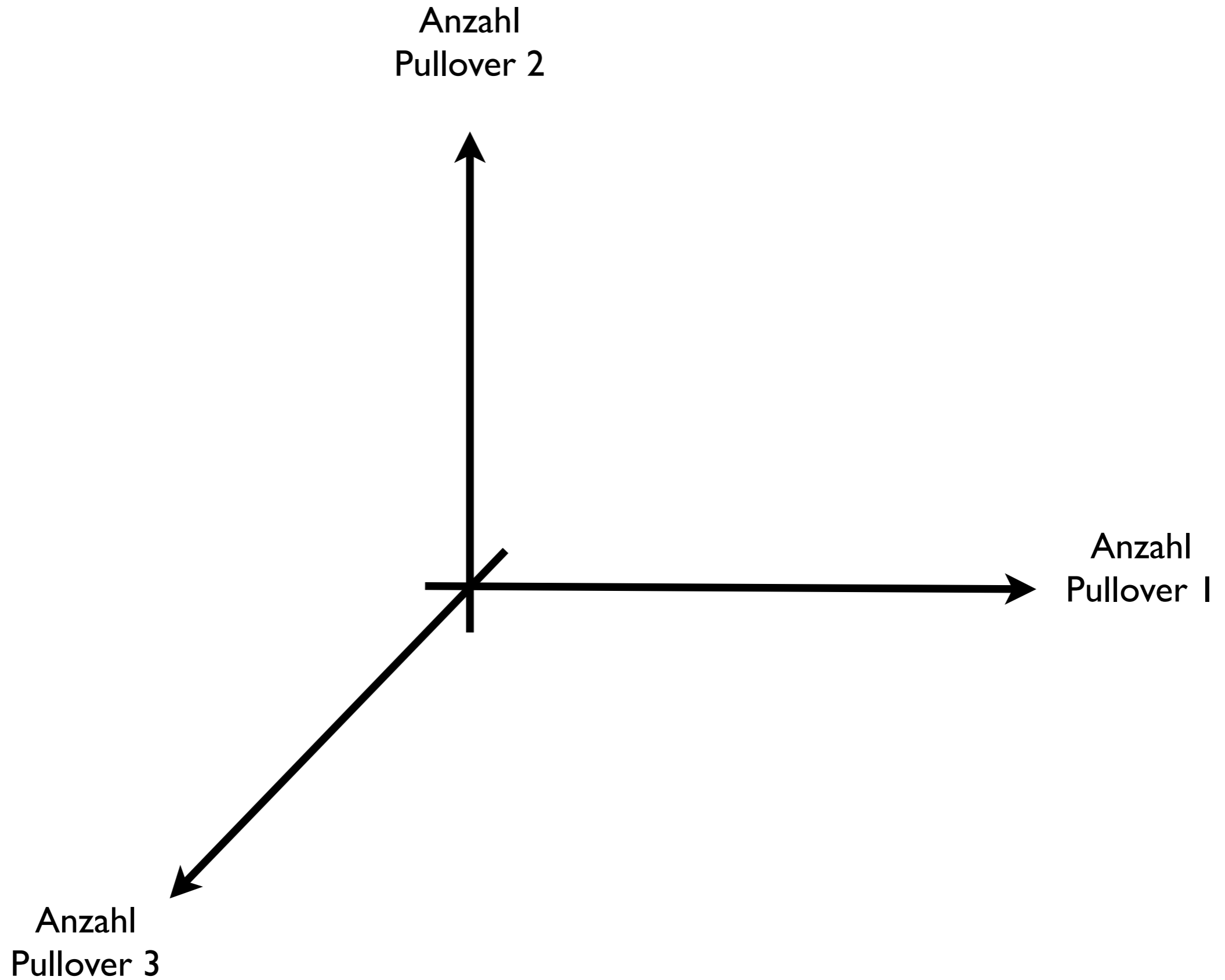
# Expansion

Anzahl  
Pullover 2

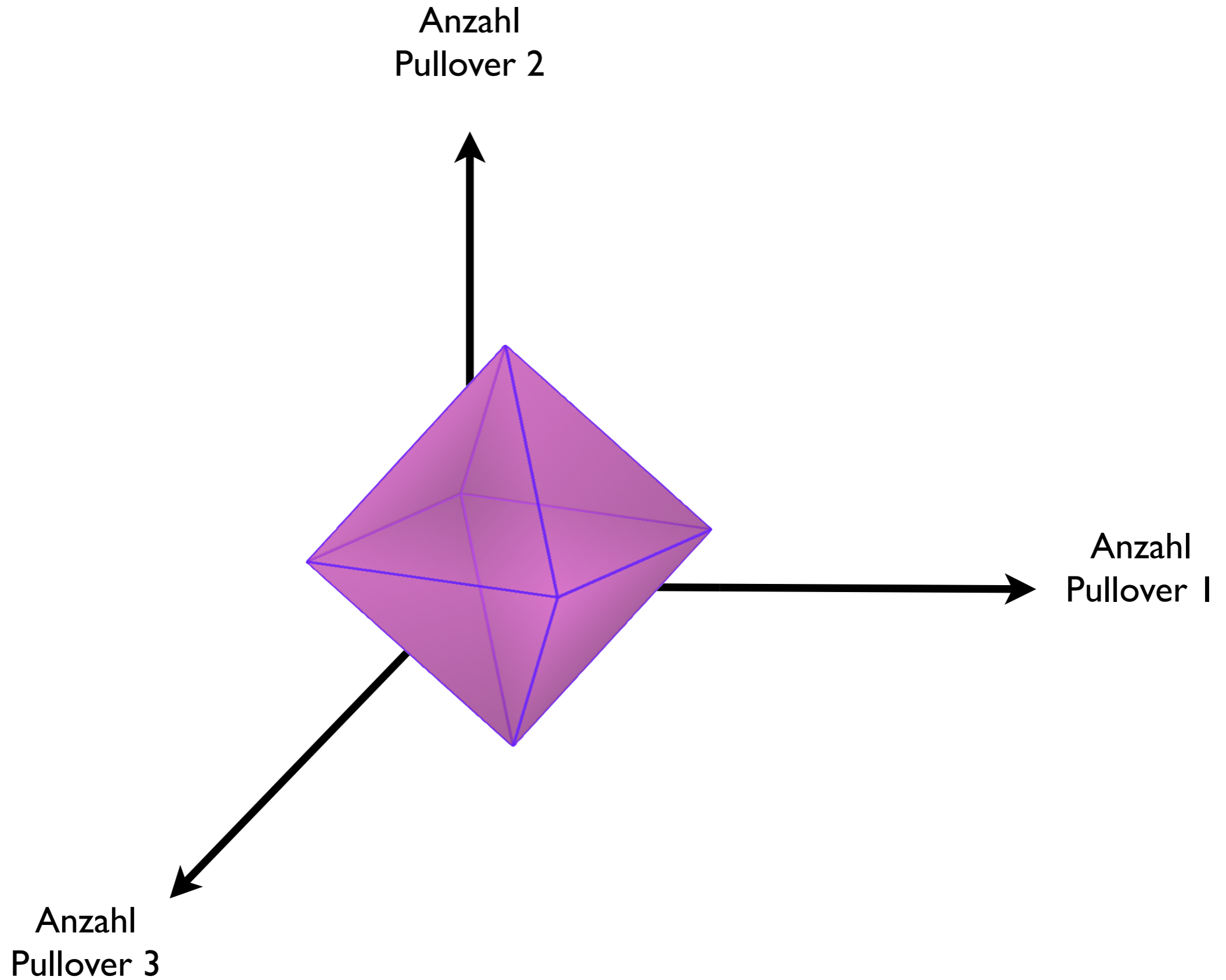


Anzahl  
Pullover 1

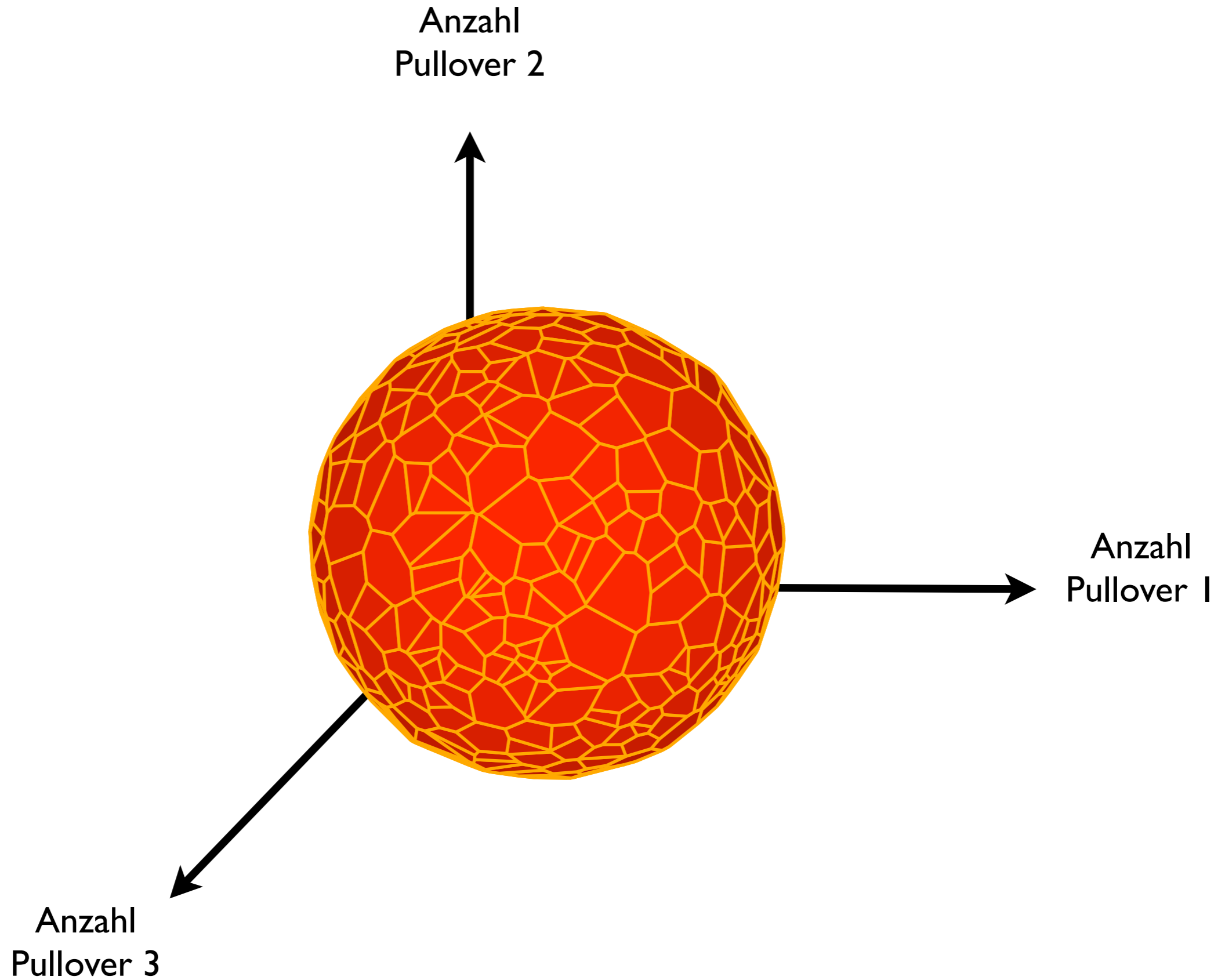
# Expansion



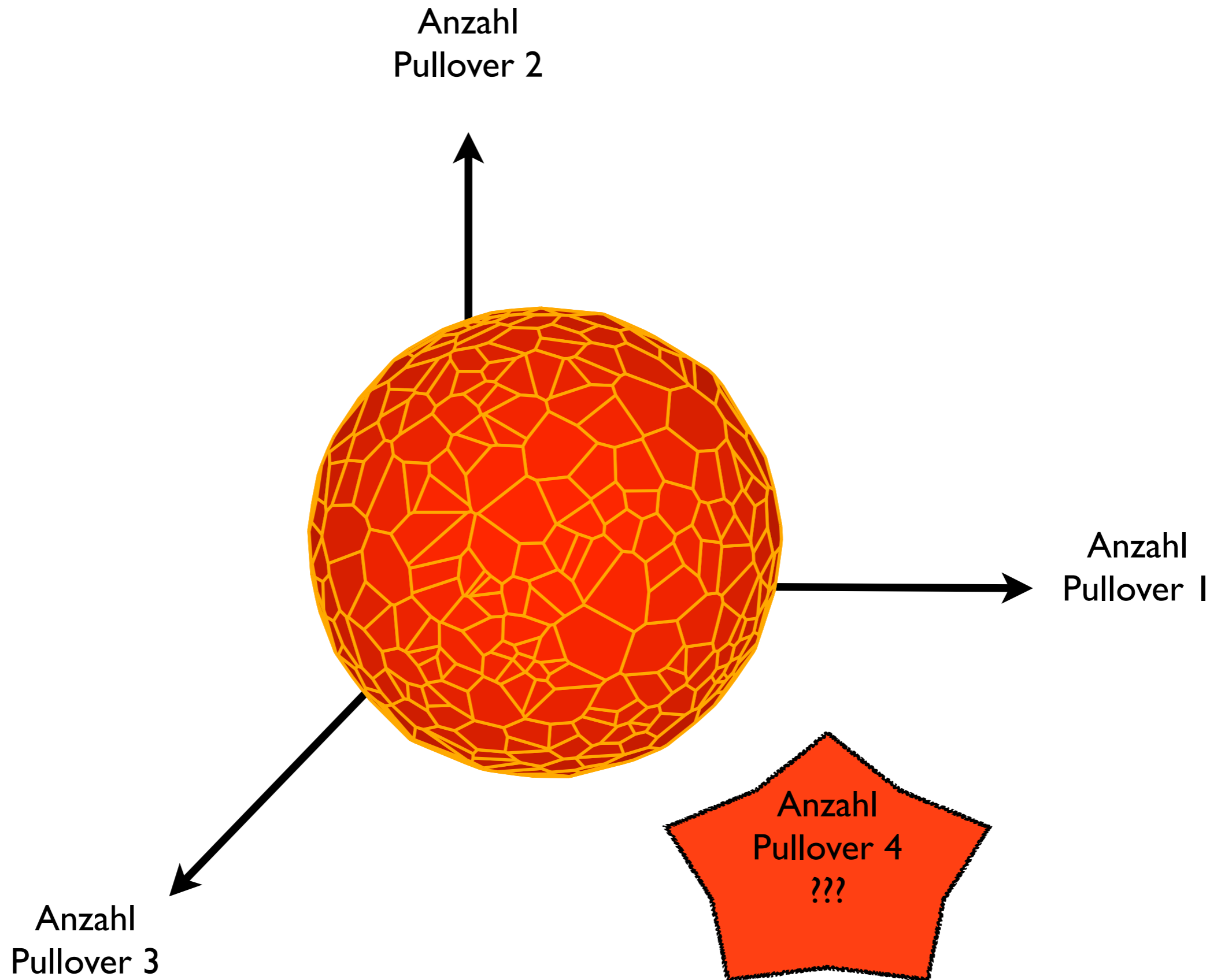
# Expansion



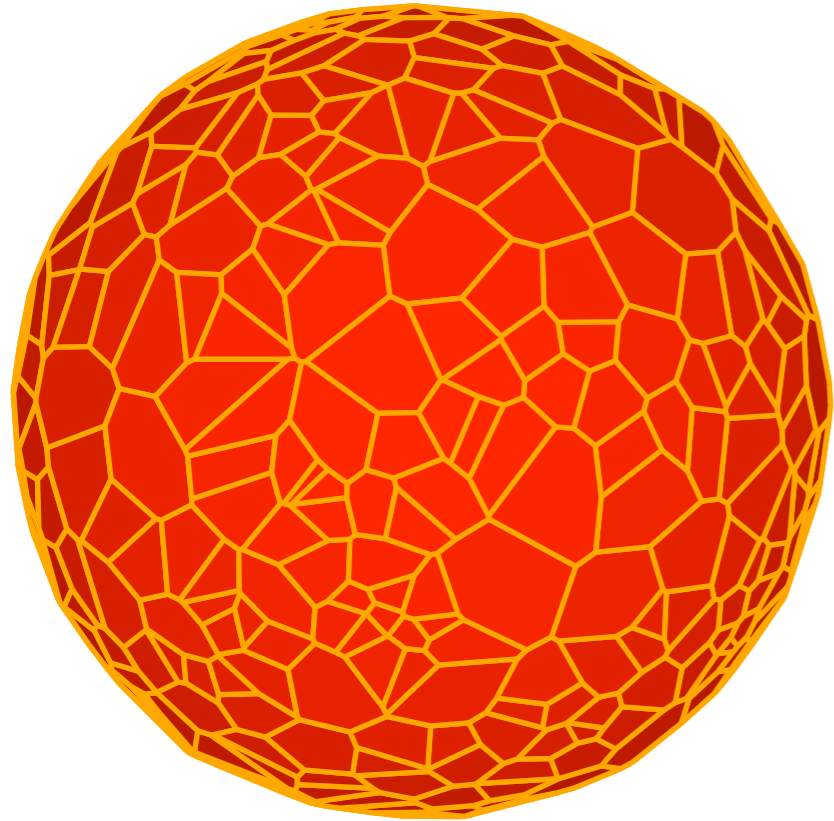
# Expansion



# Expansion

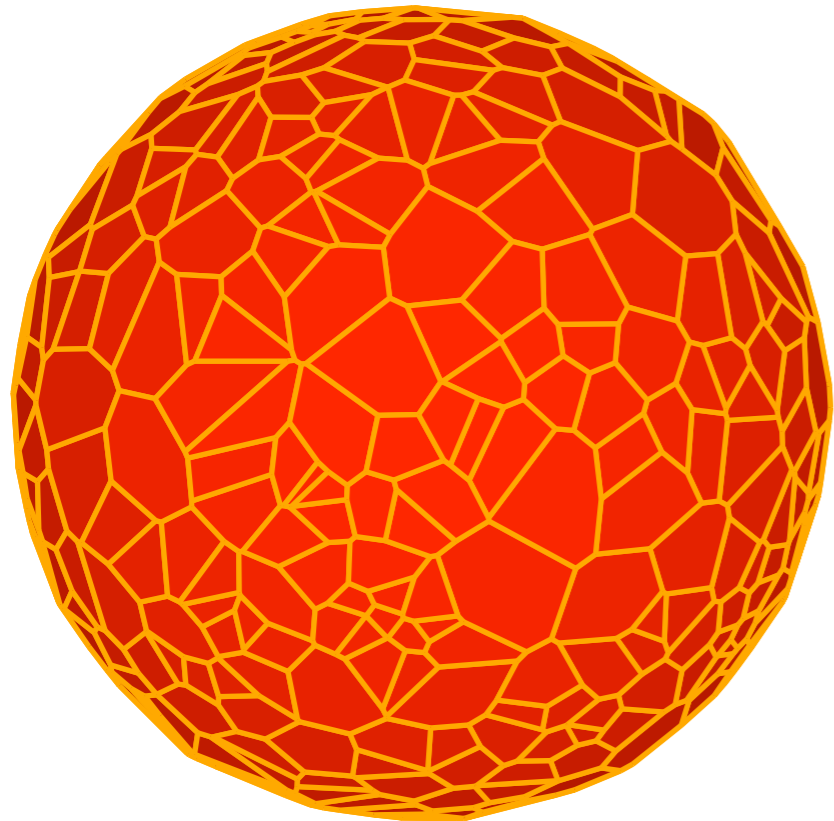


# Kombinatorische Explosion



<b>Variablen</b>	<b>Ungleichungen</b>	<b>max Ecken</b>
3	9	12
4	12	54
5	15	132
6	18	546
10	30	63 756
15	45	20 590 944
20	60	12 326 733 804
100	300	$1,61 \cdot 10^{53}$

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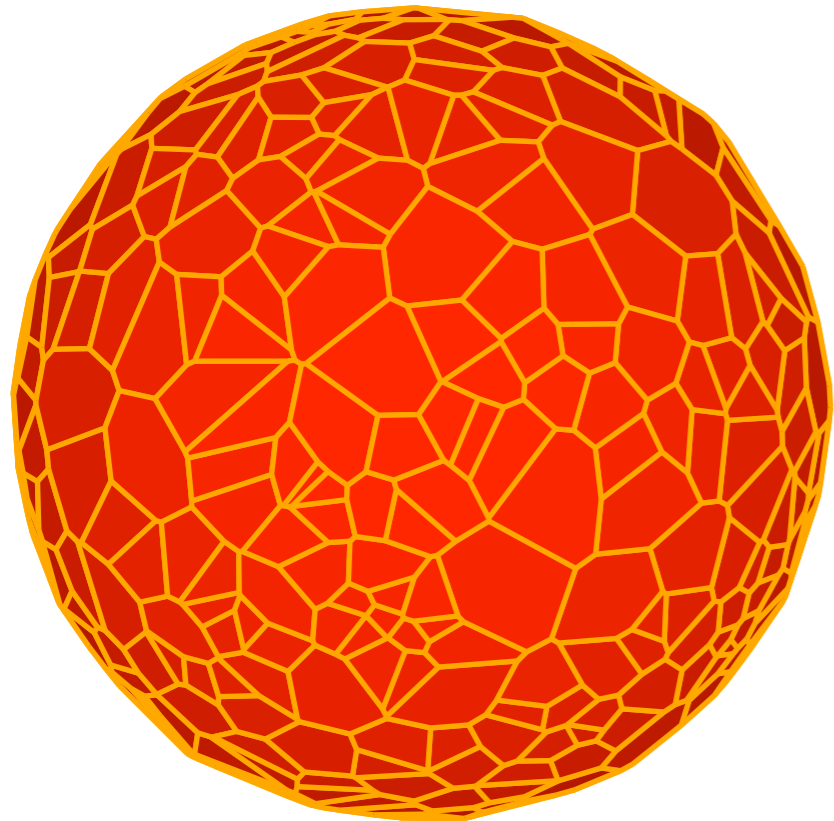


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**Wir brauchen Mathematik!**



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**Wir brauchen Mathematik!**

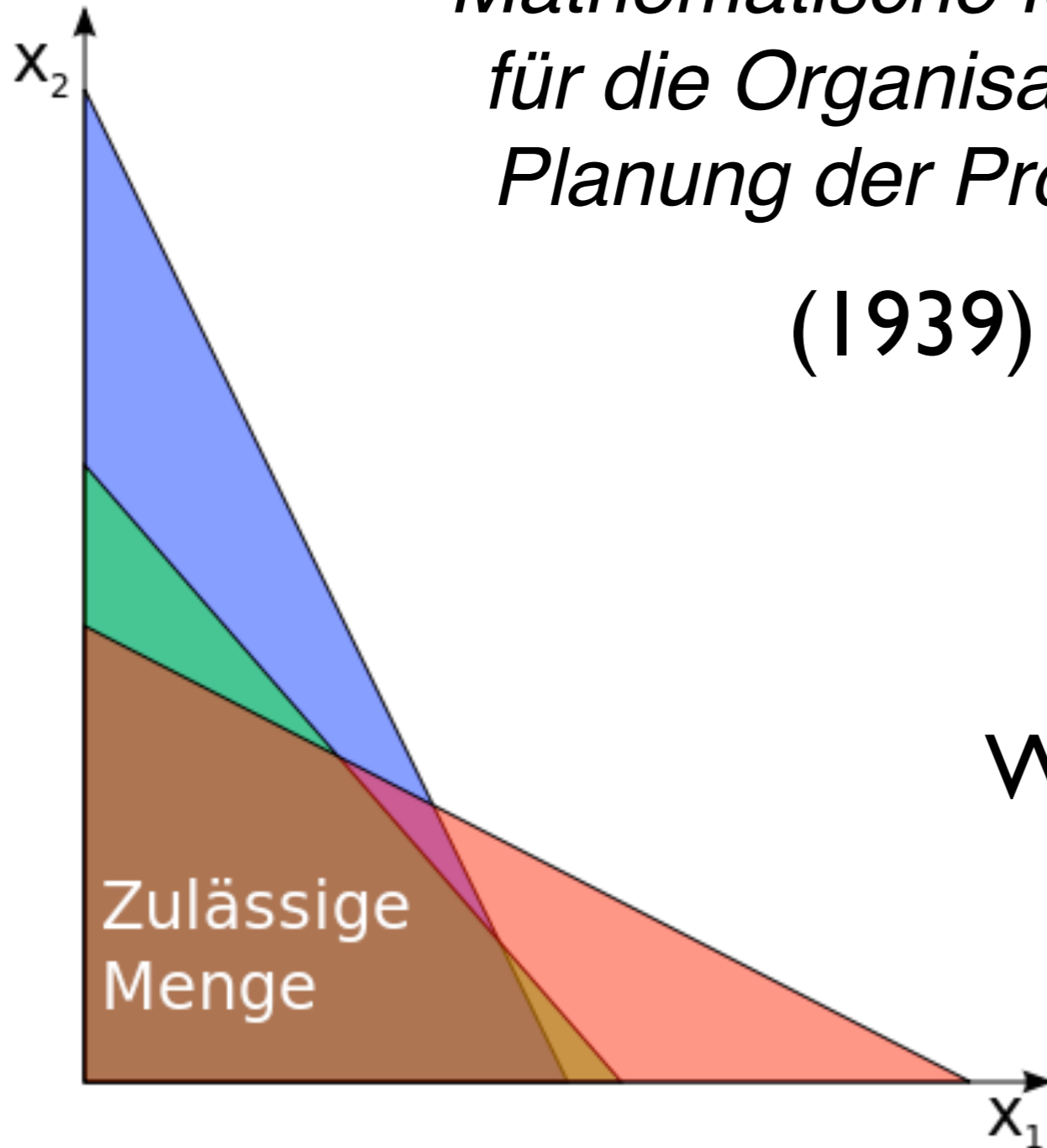
Mathematische Programmierung / Operations Research

# Lineare Programmierung (LP)

*Mathematische Methoden  
für die Organisation und  
Planung der Produktion*

(1939)

Wirtschaftsnobelpreis 1975



Leonid W.  
Kantorowitsch  
(1912-1986)

# Simplex-Algorithmus (1947)

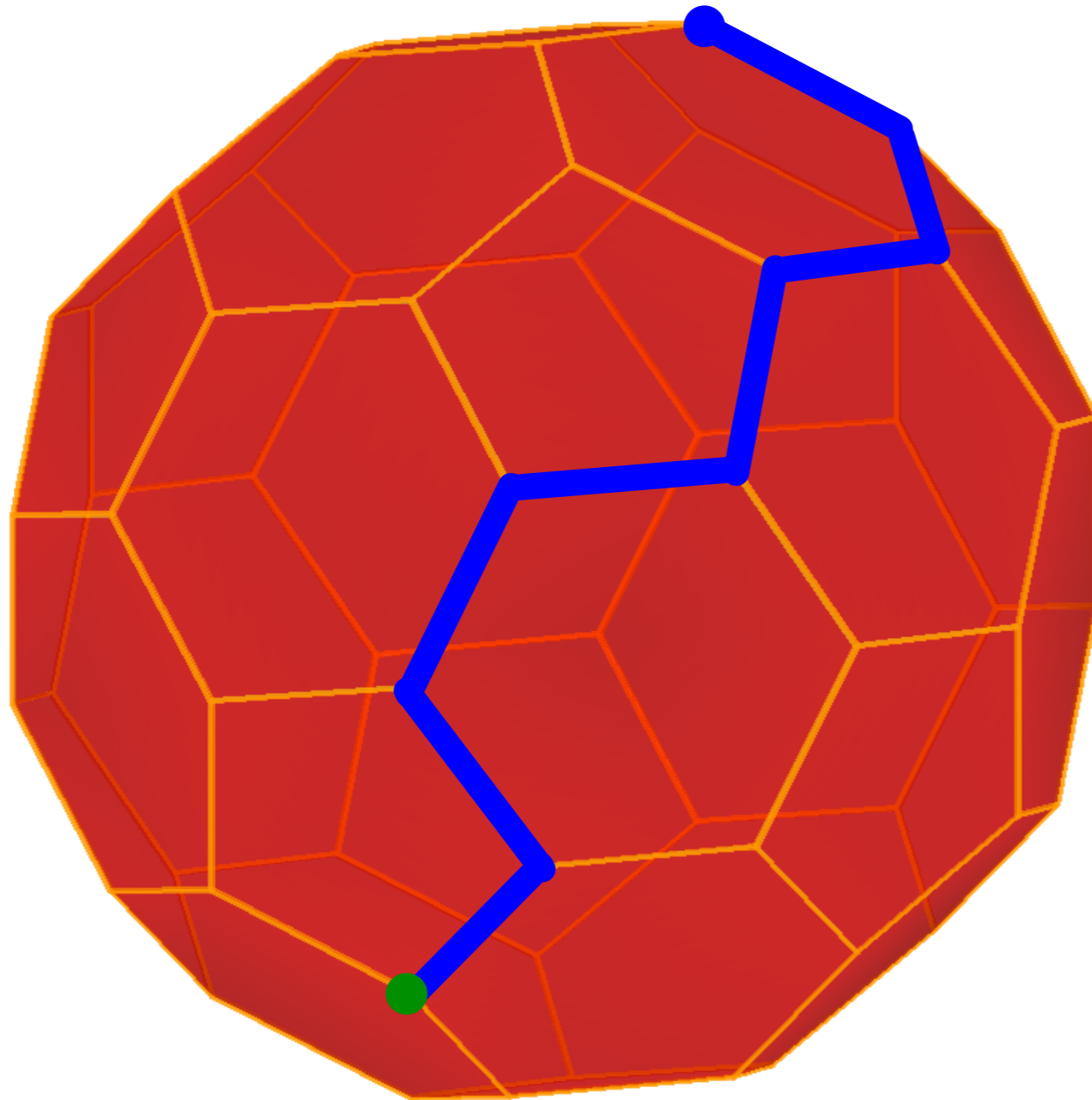


George B. Dantzig  
(1914-2005)

# Simplex-Algorithmus (1947)

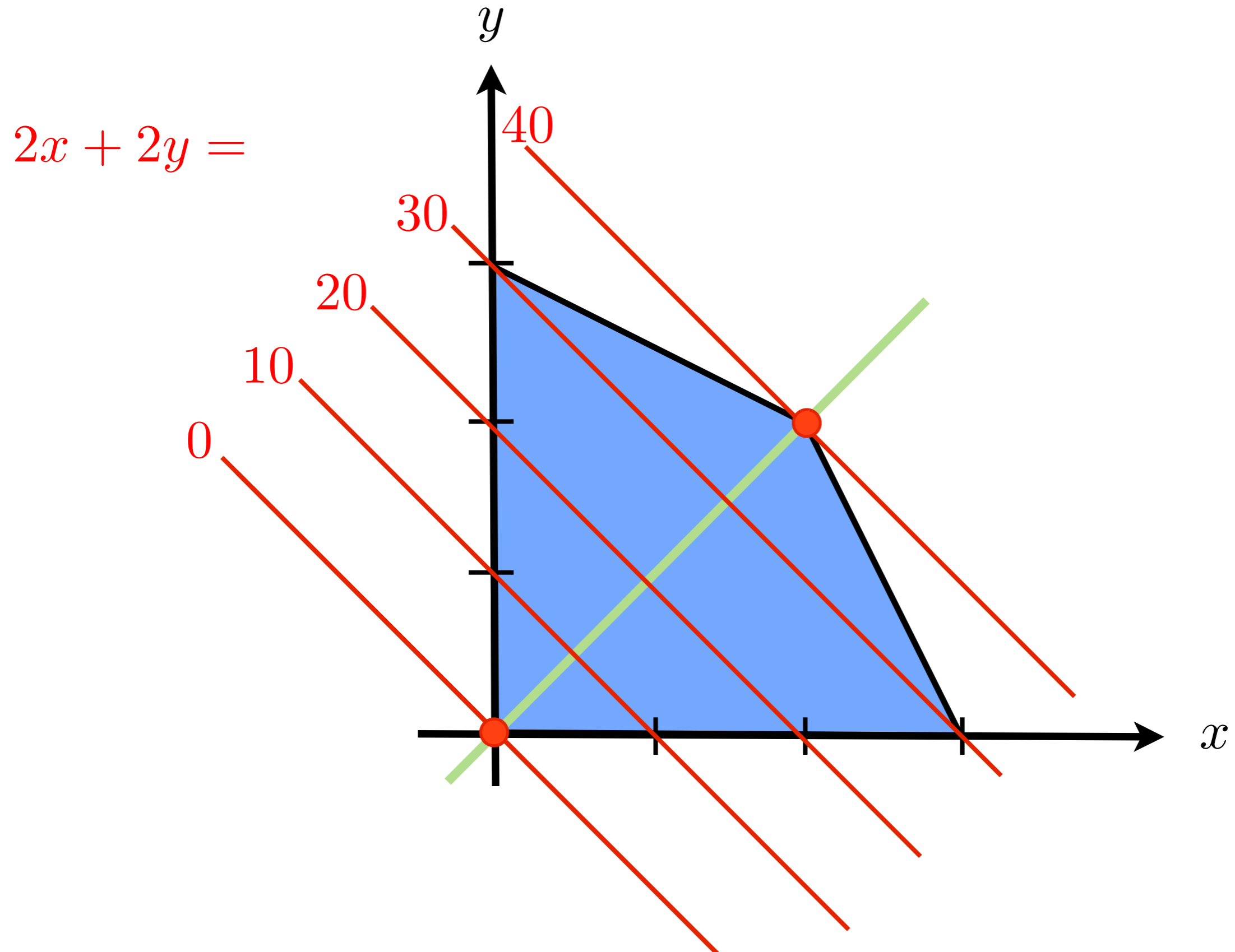


George B. Dantzig  
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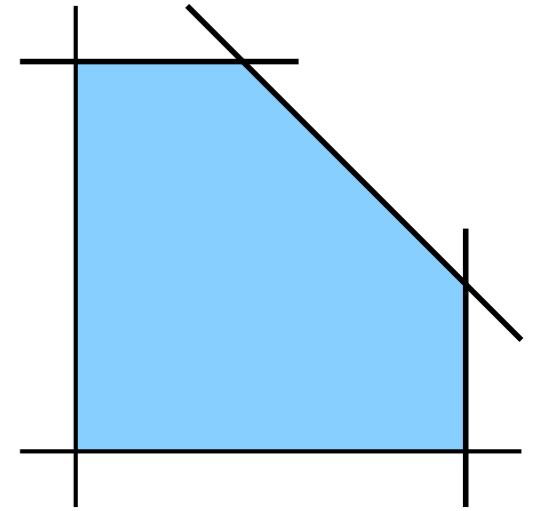


**Was kann man da noch machen?**

# Symmetrie kann man nutzen!

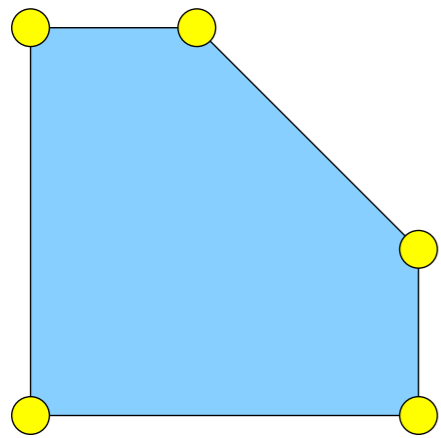
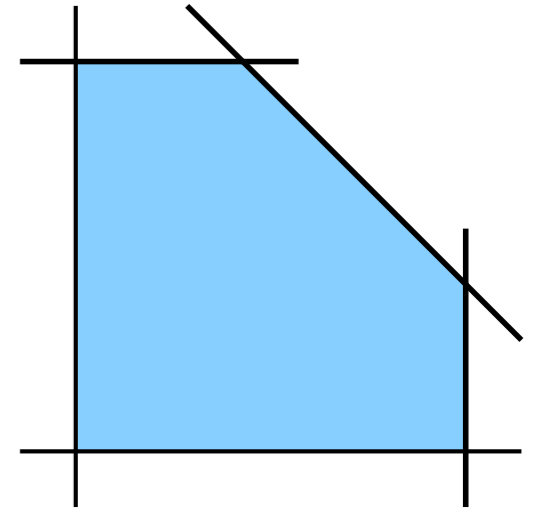


# Crucial Polyhedral Problems

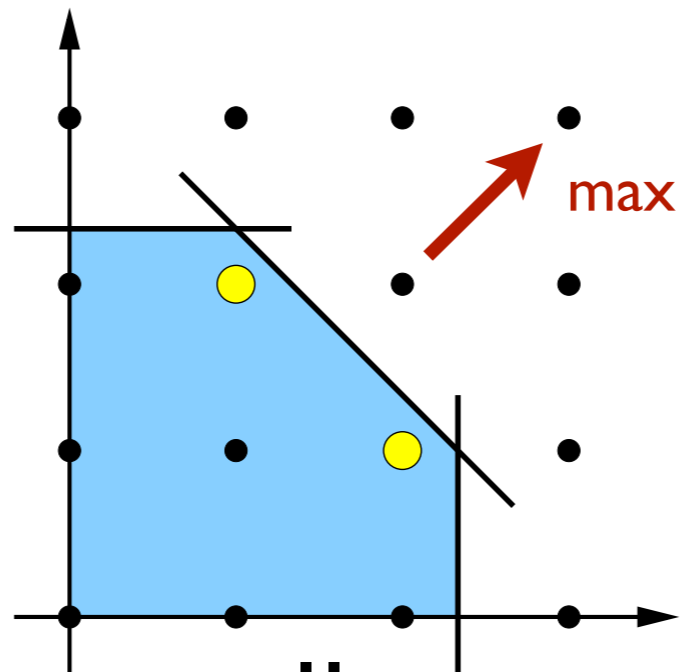


# Crucial Polyhedral Problems

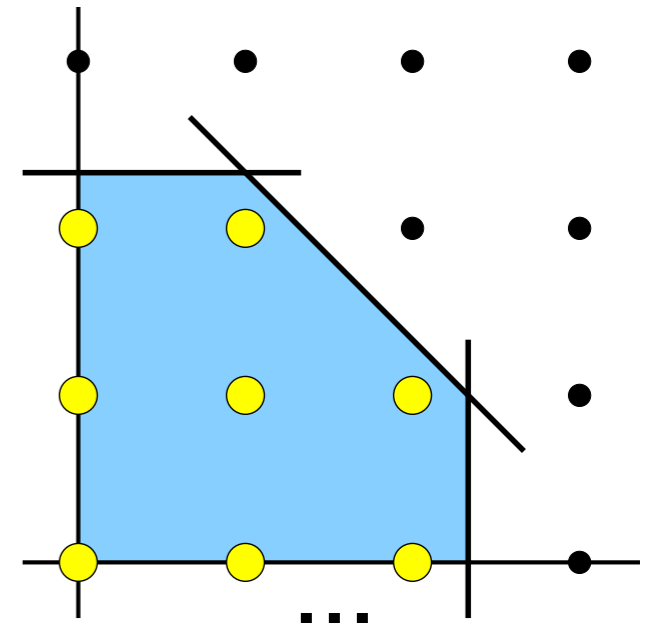
- I. Presentation Conversion
- II. Integer Linear Optimization
- III. Counting Integer Solutions



I.



II.



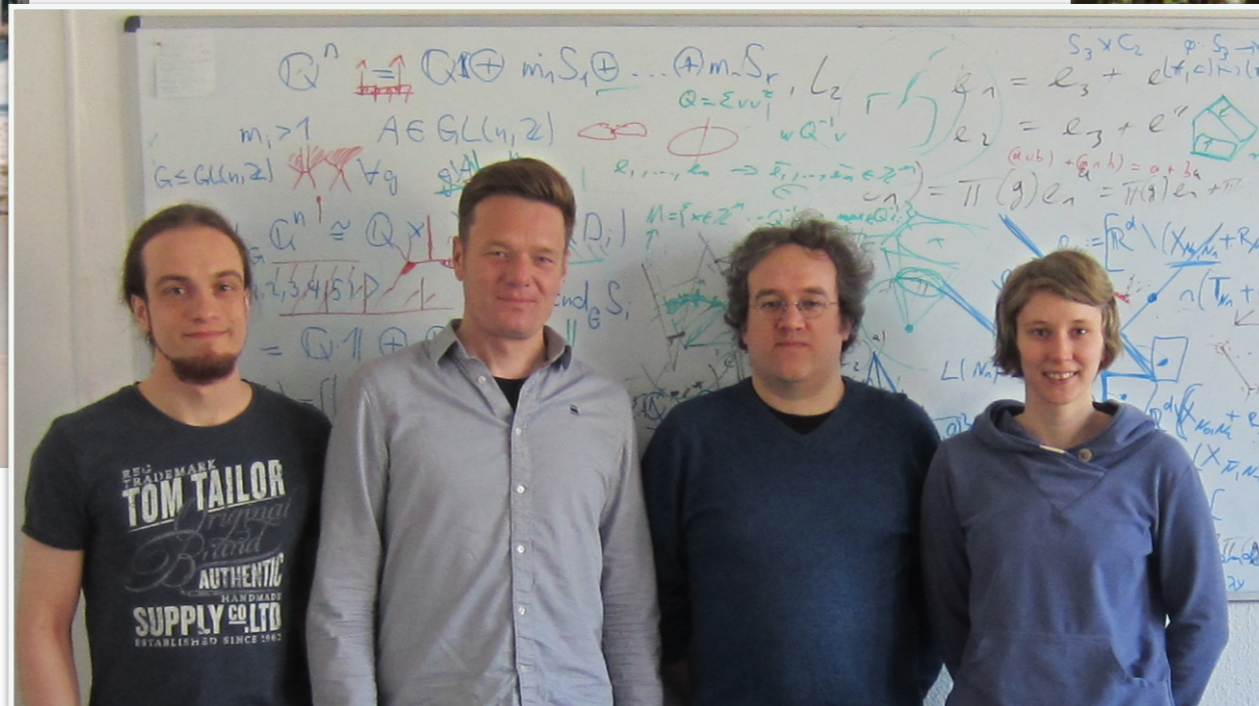
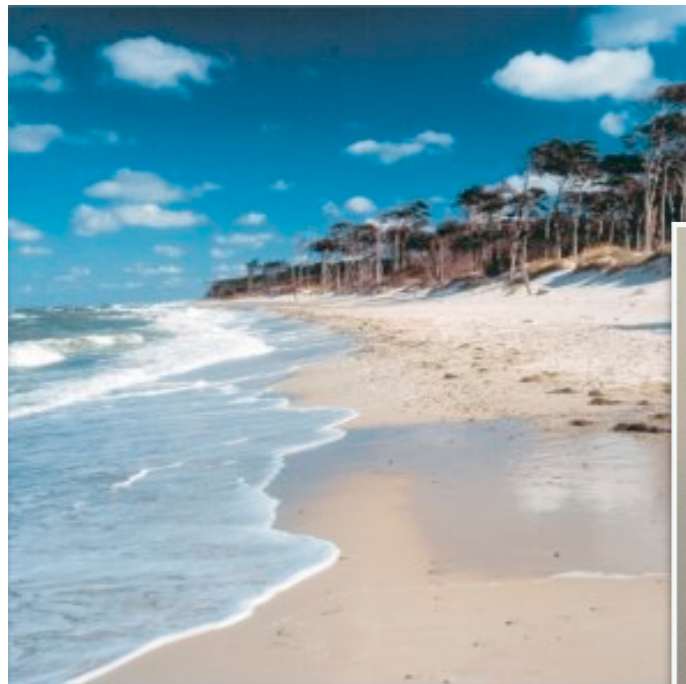
III.

**Problem: How to exploit symmetries ?**

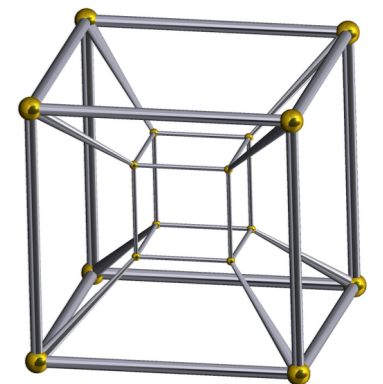
( DFG-project SCHU 1503/6-I )



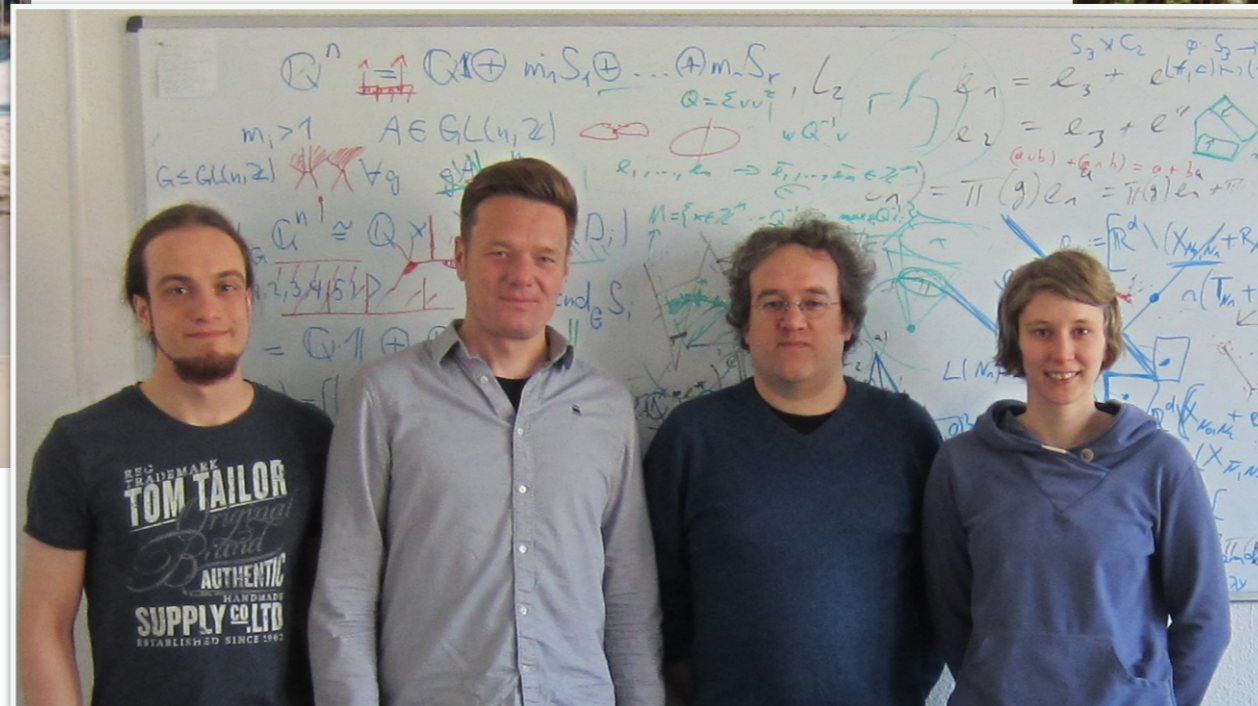
# Working on Symmetric Polyhedra



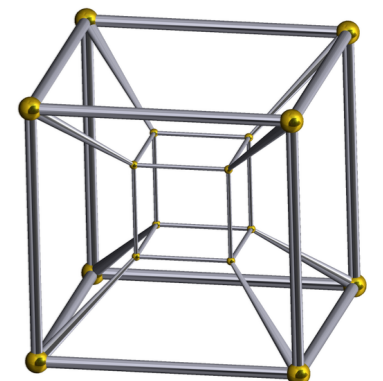
Geometry Group in Rostock



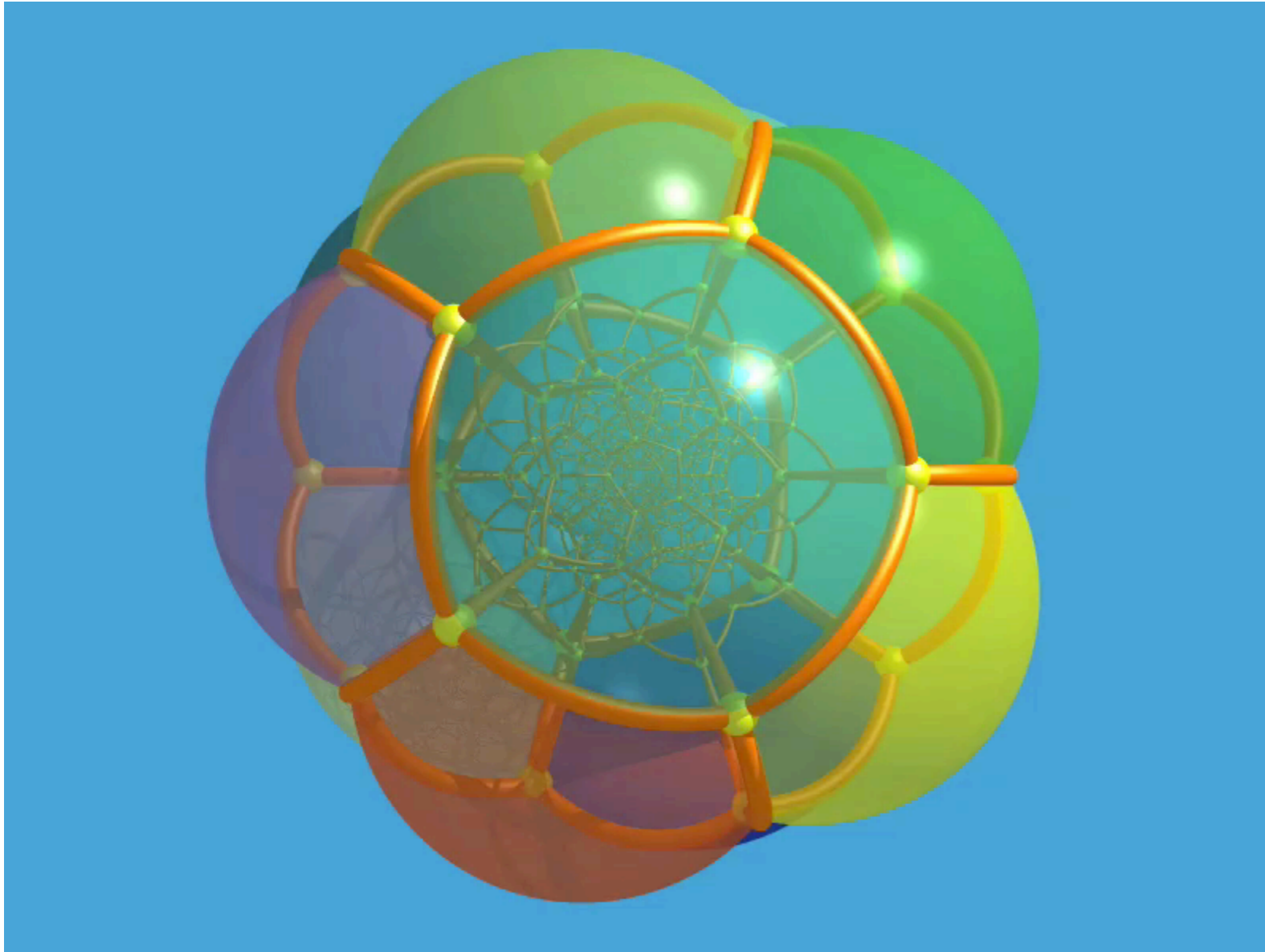
# Working on Symmetric Polyhedra



Geometry Group in Rostock  
( DFG-project SCHU 1503/6-1 )



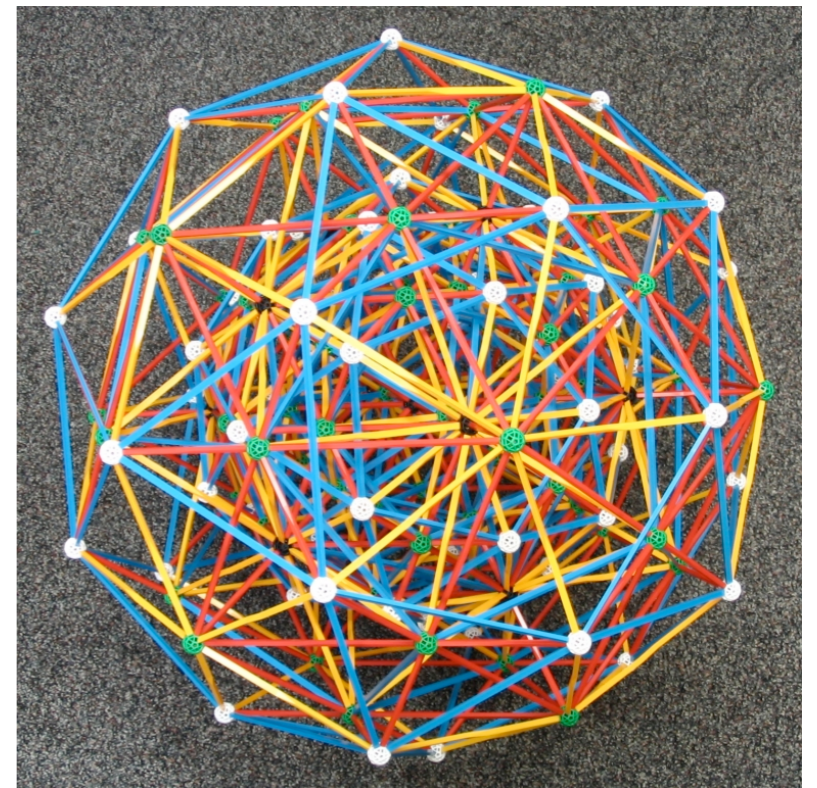
# Tipps / Links



<http://www.dimensions-math.org/>

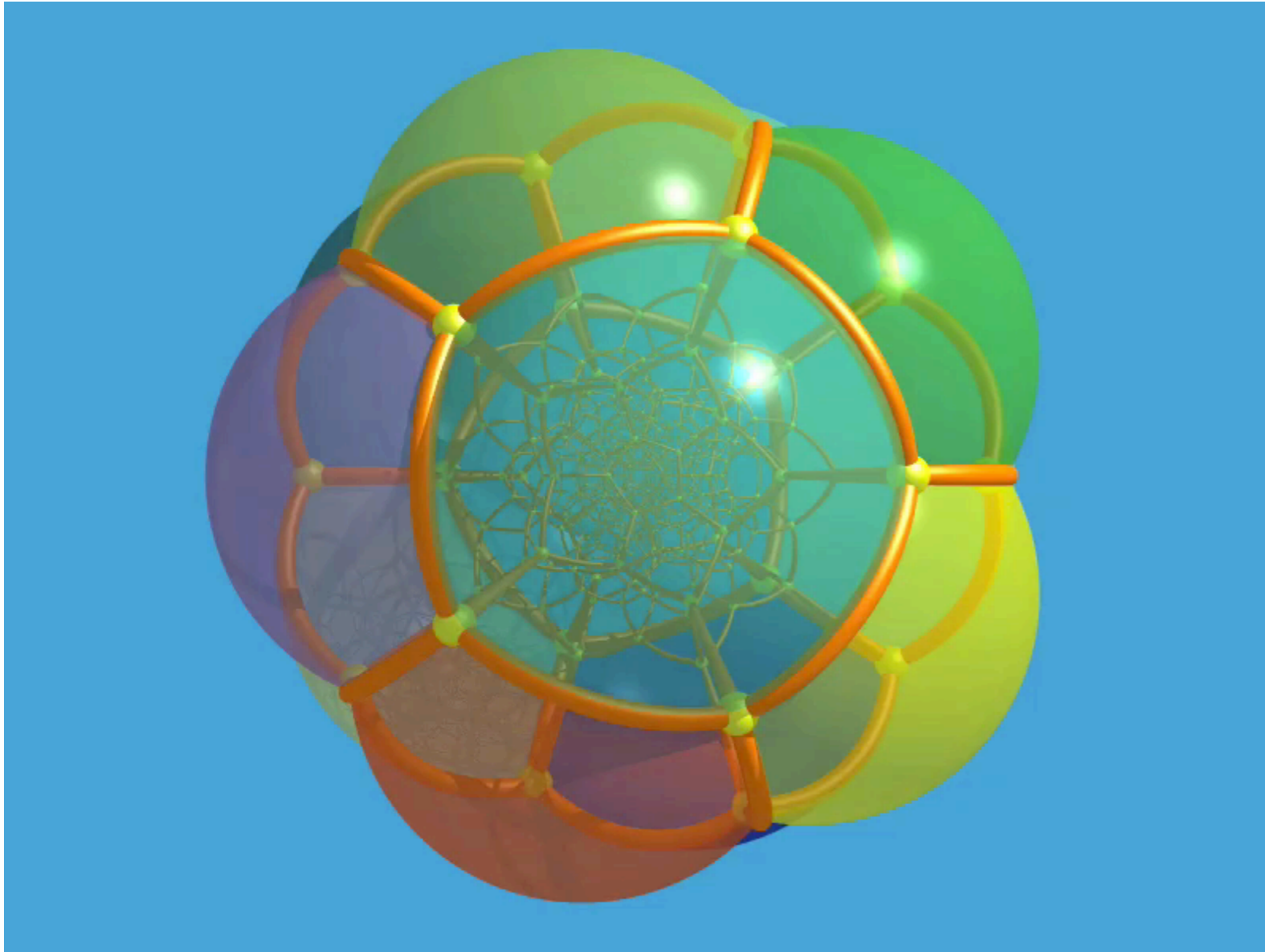


<http://www.georgehart.com/>



<http://www.geometrie.uni-rostock.de/>

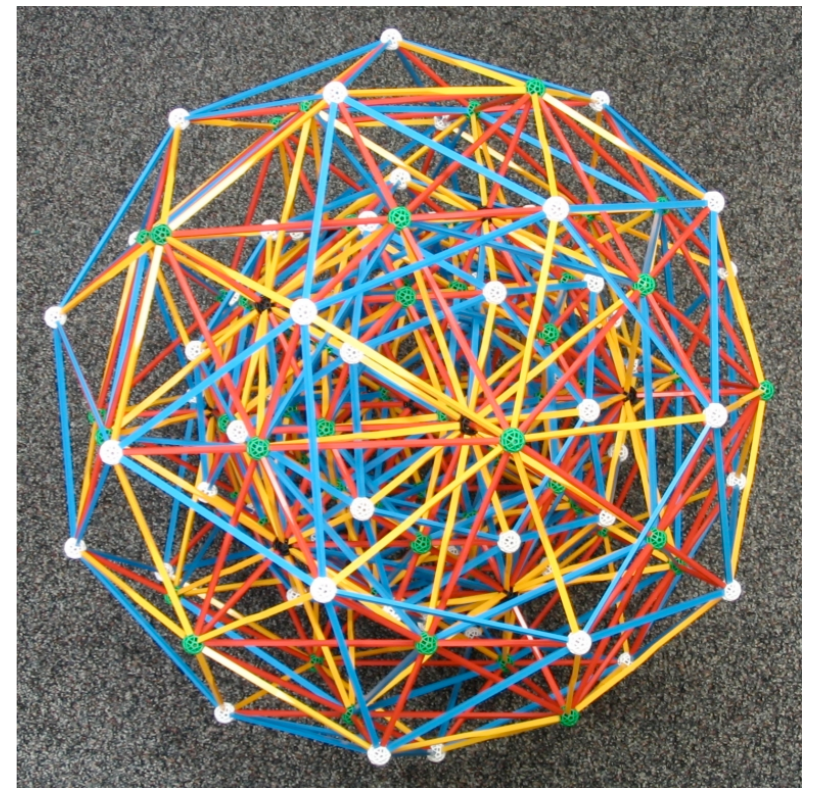
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