Hydrostatic and Stability of Floating Structures

IN A NUTSHELL

Compendium

Relevant to Questions in Exam

Robert Bronsart

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Author: Robert Bronsart
Universität Rostock
Albert-Einstein-Straße 2
18059 Rostock
Germany
email: info@schiffbaustudium.de
URL: www.schiffbauforschung.de

Acknowledgements

This document in form of a compendium is identical to chapter 14 of the corresponding text book on "*Hydrostatic and Stability of Floating Structures*". Many of the discussions are based on Heinrich Söding's work, which was first published in 1975 as Report No. 11: "Schwimmfähigkeit und Stabilität von Schiffen" of the "Lehrstuhl und Institut für Entwerfen von Schiffen und Schiffstheorie" at the University of Hannover.

Eva Binkowski, Lutz Kleinsorge, and Jonas Wagner helped to improve this document by proofreading and giving many suggestions to make the compendium clearer and more useful.

For an extended derivation and discussion of all equations including numerous practical examples, exercises, questionnaires and for a list of symbols please refer to the text book.

Request for Comments

Please help us to improve future editions by reporting any errors, inaccuracies, misleading or confusing statements and typos. Please let us also know what can be done to make this compendium more useful to you and your colleagues. We take your comments seriously and will try to incorporate reasonable suggestions into future versions.

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Hydrostatic and Stability of Floating Structures

1 Archimedes' Law

For a *freely floating* object, the following equation – representing the Archimedes' Law – holds:

$$G = \rho \cdot g \cdot \nabla = \gamma \cdot \nabla = B \quad [N] \tag{1}$$

The gravity G is equal to the buoyancy B! The mass displacement $\Delta = G/g$ becomes (with ∇ the displaced volume of the object):

$$\Delta = \rho \cdot \nabla \quad [t] \tag{2}$$

For the object to be in a state of equilibrium, the moments due to the gravitational force of the object's mass and the buoyancy must be balanced:

$$\sum \text{moments about } \eta \text{ axis} = -M_{G\xi} + M_{B\xi} = 0 = -G \cdot \xi_G + B \cdot \xi_B \qquad \curvearrowright \qquad \xi_G = \xi_B \quad [m]$$
$$\sum \text{moments about } \xi \text{ axis} = -M_{G\eta} + M_{B\eta} = 0 = -G \cdot \eta_G + B \cdot \eta_B \qquad \curvearrowright \qquad \eta_G = \eta_B \quad [m]$$

The two coordinate axes ξ and η define a plane parallel to the fluid surface. The origin can be placed anywhere in space and must not necessarily lie e.g. in the water plane or at a specific point of the object.

For properties specified by an index: the index indicates the direction of the lever, not the reference axis! Examples: δR_y is the resulting moment about the x-axis, $M_{w\xi}$ is the moment of the water plane area about the η-axis, I_{xs} is the moment of inertia of the water plane area about the y-axis which passes through the centroid of the water plane LCF.

2 Small Changes in Floating Position: Buoyancy and Gravity

The three relevant degrees of freedom (DOF) in hydrostatic and stability calculations are:

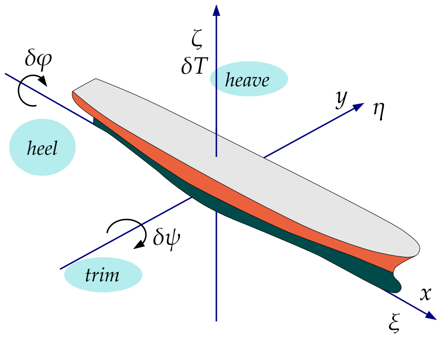


Figure 1: Ship changes in floating position: 3 DOF

For an equilibrium floating condition the force (R_ζ) and moments (δR_ξ and δR_η) due to **small changes** in any combination of the three DOF (δT, $\delta\psi$, $\delta\varphi$) can be calculated as:

$$\begin{pmatrix} \delta R_\zeta \\ \delta R_\xi \\ \delta R_\eta \end{pmatrix} = \begin{bmatrix} \gamma \cdot A_w & \gamma \cdot M_{w\xi} & \gamma \cdot M_{w\eta} \\ \gamma \cdot M_{w\xi} & G \cdot (\frac{I_\xi}{\nabla} + \zeta_\nabla - \zeta_G) & G \cdot \frac{I_{\xi\eta}}{\nabla} \\ \gamma \cdot M_{w\eta} & G \cdot \frac{I_{\xi\eta}}{\nabla} & G \cdot (\frac{I_\eta}{\nabla} + \zeta_\nabla - \zeta_G) \end{bmatrix} \cdot \begin{pmatrix} \delta T \\ \delta\psi \\ \delta\varphi \end{pmatrix} \tag{3}$$

Small changes only apply if: $\{\sin, \tan\}\{\delta\varphi, \delta\psi\} \approx \{\delta\varphi, \delta\psi\}$; $\cos\{\delta\varphi, \delta\psi\} \approx 1$ **and** matrix elements (hull form parameter!) can be assumed constant over $\delta T, \delta\psi, \delta\varphi$!

For symmetrically shaped ships ($M_{w\eta} = I_{\xi\eta} = 0$) and a coordinate system identical to the **principle coordinate axes of the water plane** (origin at LCF ($x_w = 0$); x-axis in centre line; y-axis to port side; z-axis upwards) the three equations in (3) simplify to:

$$\delta R_z = \gamma \cdot A_w \cdot \delta T \qquad [N] \tag{4}$$

$$\delta R_x = G \cdot (\frac{I_{xs}}{\nabla} + z_\nabla - z_G) \cdot \delta\psi \qquad [Nm] \tag{5}$$

$$\delta R_y = G \cdot (\frac{I_{ys}}{\nabla} + z_\nabla - z_G) \cdot \delta\varphi \qquad [Nm] \tag{6}$$

Making use of naval architectural notations the two latter equations can be rewritten as:

$$\delta R_x = G(BM_L + KB - KG) \cdot \delta\psi = G \cdot GM_L \cdot \delta\psi \qquad [Nm] \tag{7}$$

$$\delta R_y = G(BM + KB - KG) \cdot \delta\varphi = G \cdot GM \cdot \delta\varphi \qquad [Nm] \tag{8}$$

3 Trim

A positive trim δt is given if the local draught at the bow is larger than at the stern. The resulting moment about the transverse principal axis (through LCF!) can be written as:

$$\delta\psi \approx \frac{\delta t}{L_A} \quad \curvearrowright \quad \delta R_x = G \cdot GM_L \cdot \delta\psi = G \cdot GM_L \cdot \frac{\delta t}{L_A} \tag{9}$$

With δt set to 1, the "moment to alter trim one metre" becomes (with $BM_L \gg KG \approx KB$):

$$E_T = \frac{\Delta \cdot GM_L}{L_A} = \frac{\Delta}{L_A} \cdot (KB + BM_L - KG) \approx \frac{\Delta}{L_A} \cdot BM_L = \frac{\Delta}{L_A} \cdot \frac{I_{xs}}{\nabla} = \rho \cdot \frac{I_{xs}}{L_A} \quad [t] \tag{10}$$

4 Stability Criteria

Three equations (1 – 3) and three inequalities (4 – 6) have to be fulfilled for every floating position in a **stable equilibrium state**:

$$\begin{array}{ll}
(\mathbf{1}) & G = \rho \cdot g \cdot \nabla \\
(\mathbf{2}) & B \cdot \xi_B = G \cdot \xi_G \;\curvearrowright\; \xi_B = \xi_G \\
(\mathbf{3}) & B \cdot \eta_B = G \cdot \eta_G \;\curvearrowright\; \eta_B = \eta_G \\
(\mathbf{4}) & A_w > 0 \\
(\mathbf{5}) & \frac{I_{\xi s}}{\nabla} + \zeta_\nabla - \zeta_G > 0 \\
(\mathbf{6}) & \frac{I_{\eta s}}{\nabla} + \zeta_\nabla - \zeta_G - \frac{(\frac{I_{\xi\eta s}}{\nabla})^2}{\frac{I_{\xi s}}{\nabla} + \zeta_\nabla - \zeta_G} > 0
\end{array}$$

For symmetrically shaped ships ($I_{\xi\eta s} \stackrel{!}{=} 0$) the two inequalities (5) and (6) can be rewritten in naval architectural notation:

$$(\mathbf{5}) \quad \curvearrowright GM_L = KB + BM_L - KG > 0 \tag{11}$$

$$(\mathbf{6}) \quad \curvearrowright \mathbf{GM} = \mathbf{KB} + \mathbf{BM} - \mathbf{KG} > \mathbf{0} \tag{12}$$

GM and GM_L are defined through the ship hull form, the light ship weight distribution and in ship operation through the actual loading condition ($KB, KG, BM = \frac{I_{\eta s}}{\nabla}, BM_L = \frac{I_{\xi s}}{\nabla}$): $\{GM, GM_L\} = f$(hull form, loading condition, vertical centroid light ship weight). In presence of fluid free surfaces in partially filled tanks (ρ_T, I_{Fys}) GM has to be corrected (GM_R):

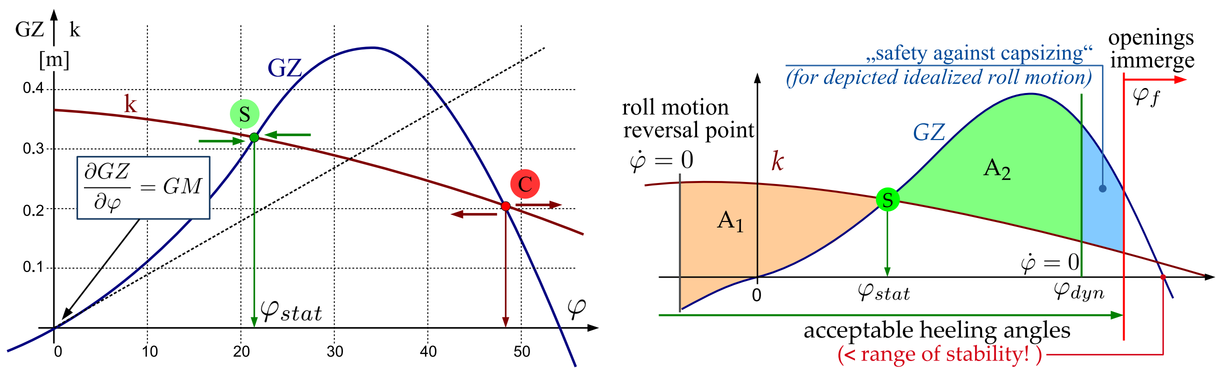
$$GM_{corr} = \sum \frac{\rho_T}{\rho_S} \cdot \frac{I_{Fys}}{\nabla}$$

$$\curvearrowright \quad GM_R = KB + \frac{I_T - \sum \frac{\rho_T}{\rho_S} \cdot I_{Fys}}{\nabla} - KG \tag{13}$$

The link between the two approaches "small" $\varphi \to 0$ and "any" φ: The slope of the righting arm curve at the upright floating position ($\varphi = 0$) is equal to GM, see also Equation 21.

$$\left.\frac{\partial(GZ)}{\partial\varphi}\right|_{\varphi=0} = GM \stackrel{!}{>} 0 \tag{14}$$

Stability criteria for a floating position under an upsetting moment (M_k) expressed by the upsetting arm $k = M_k/B$: note the fundamental difference between intersections "S" ($\to$ stable)



(a) Equilibrium angel of heel: φ_{stat} (b) Dynamic stability criterion

Figure 2: Stability criteria – righting arm curve

and "C" ($\rightarrow$ unstable) shown in Figure 2 (a), yielding the stability criteria:

$$\left.\frac{\partial(GZ)}{\partial\varphi}\right|_{\varphi=\varphi_{stat}} \overset{!}{>} \left.\frac{\partial k}{\partial\varphi}\right|_{\varphi=\varphi_{stat}} \tag{15}$$

Dynamic stability criteria (energy balance very simplified method): comparison of areas between upsetting (k) and uprighting (GZ) arm curves, neglecting any damping effects, see Figure 2 (b):

$$\int_{\dot{\varphi}=0}^{\varphi_{stat}} (k - GZ)\, d\varphi = \int_{\varphi_{stat}}^{\varphi_{dyn}} (GZ - k)\, d\varphi \quad \curvearrowright \quad A_1 = A_2 \tag{16}$$

The upper boundary for the integration φ_{dyn} (at which again $\dot{\varphi} = 0 \rightarrow$ roll motion reversal point) must lie within the range of stability and has to be less than the angle of downflooding (φ_f) preventing progressive flooding, see also Figure 7. Damping (energy dissipation) due to generated waves, friction and vortexes (e.g. by bilge keels) results in A_2 is smaller than A_1.

5 Inclining Test

The results of an inclining test are: 1) the actual KG in light ship condition and 2) the maximum deadweight (considering maximum draught and freeboard).

$$\delta R_y = G \cdot GM \cdot \delta\varphi = G \cdot (KB + BM - KG) \cdot \delta\varphi \qquad \text{(see equations } 3 \rightarrow 6 \rightarrow 8\text{)}$$
$$= \gamma \cdot \nabla \cdot (KB + BM - KG) \cdot \delta\varphi$$
$$\downarrow$$
$$KG = \underbrace{KB + BM - \frac{1}{\nabla}}_{\text{to be calculated}} \cdot \underbrace{\frac{1}{\gamma(= \rho \cdot g)}}_{\text{actual value for } \rho!} \cdot \underbrace{\frac{\delta R_y}{\delta\varphi}}_{\text{from measurements}} \tag{17}$$

The accuracy of the KG value is essential for stability calculations (GM and righting arm GZ), the maximum deadweight verification is essential for the contract fulfilment.

6 Cross Curves of Stability

$$R_\eta = B \cdot \eta_B - G \cdot \eta_G = G(\eta_B - \eta_G) = G(KN - \eta_G) \tag{18}$$

η_B (often called "KN") is the transverse location of the centroid of displaced volume measured in a coordinate system parallel to the fluid surface with an origin at the intersection of the centre line ℄ and the base line BL at (K), see Figure 3. The function of KN for a constant angle of heel φ but a varying displaced volume ∇ is called "Cross Curve of Stability".

Very much simplified calculation based on Wall Side Formula. Factor λ ($\sim 0.6\ldots1$) expresses the effect of a hull form not being wall sided:

$$KN = (KM + \underbrace{\lambda \cdot \frac{1}{2}\tan^2\varphi \cdot BM}_{\textit{form effect}}) \cdot \sin\varphi \tag{19}$$

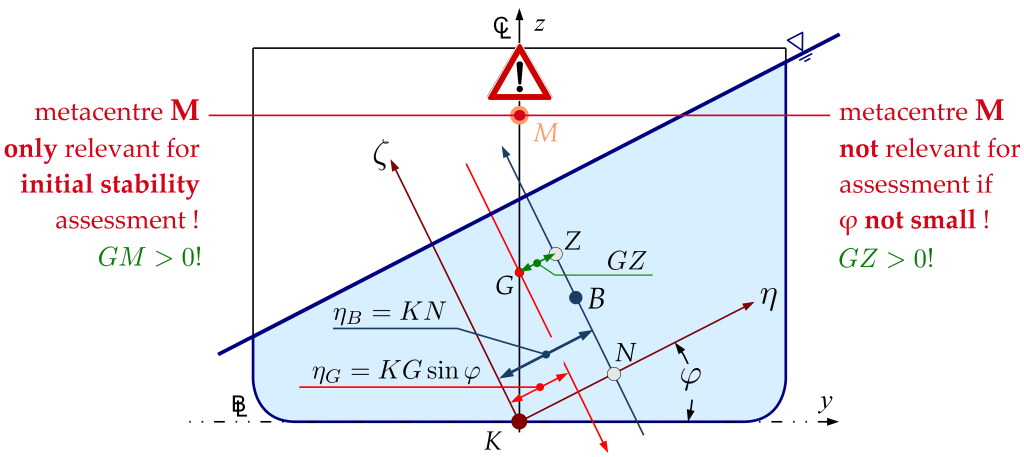


Figure 3: Vessel with larger angle of heel: location of η_G, $\eta_B = KN$ and distance GZ

7 Righting Arm Curve

Two moments due to gravity and buoyancy yield the righting arm: the distance measured parallel to the water plane between the two vectors of the gravity and buoyancy, see Figure 3.

$$GZ = \eta_B - \eta_G = KN - \eta_G$$

The transverse location of the centroid of mass (η_G):

$$\eta_G = y_G \cos\varphi + z_G \sin\varphi = y_G \cos\varphi + KG \sin\varphi$$

$$\curvearrowright$$

$$R_\eta = G \cdot GZ = G \cdot (KN - \eta_G) = G \cdot \underbrace{(KN - KG\sin\varphi)}_{GZ} - \underbrace{G \cdot y_G \cos\varphi}_{y_G \neq 0}$$

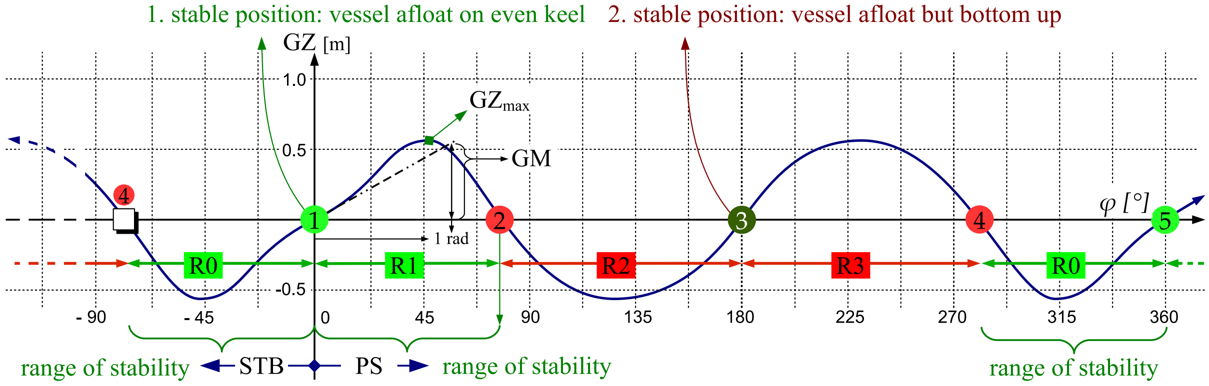


Figure 4: Righting arm curve over a full range of φ

With a symmetrical mass distribution ($\rightarrow y_G = 0$) the righting arm $GZ(\varphi)$ is, see Figure 3:

$$GZ = KN - KG \sin \varphi \tag{20}$$

The righting arm curve shows the righting arm for a specific ship ($\rightarrow$ hull form) under a specified loading condition ($\rightarrow KG$, hull form and floating position $\rightarrow KN$) over the heeling angle φ. The slope of the righting arm curve at the upright floating position ($\varphi = 0$) is equal to GM. Without upsetting moments, stable floating positions are defined by (①, ③, ⑤):

$$\left.\frac{\partial(GZ)}{\partial\varphi}\right|_{\varphi=0} = GM \overset{!}{>} 0 \quad \text{yields more general} \quad \left.\frac{\partial(GZ)}{\partial\varphi}\right|_{\mathbf{GZ=0}} \overset{!}{>} 0 \tag{21}$$

The second intercept ② defines the vessel's "range of stability". The third intercept ③ is at $\varphi = 180°$ only if the ship is symmetrical to the centre plane with respect to its watertight shape (superstructures!) and the mass distribution. The two curve segments R0 between ❑ and ① and between ④ and ⑤ are identical. The floating position ① and ⑤ are identical, like ❑ and ④. At ①, ③ and ⑤ the adjacent curve segments are point reflected at these intercepts.

Righting arm curve as function of hull form (A, B) in waves (head or following sea condition):

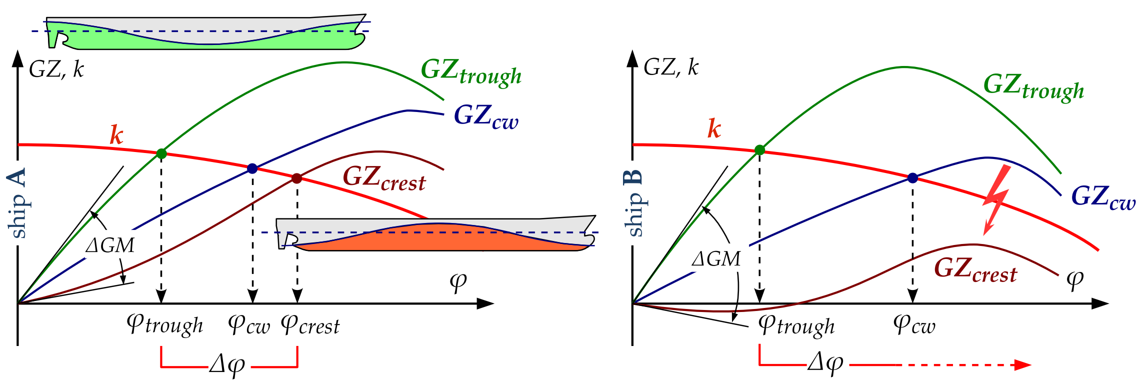


Figure 5: Heeling and righting arm curves of two hull forms in longitudinal waves

The righting arm curve becomes a function of time: fluctuating between two pronounced conditions: wave trough and crest condition, both refer to the wave height at midships ($L_{pp}/2$).

8 Heeling Moments

Main reasons for heeling moments are: transverse shift of loads, crowding of people, partially filled tanks (see additional GM reduction!), wind, waves (most important but not considered here), turning, towing and anchoring.

9 IMO Intact Stability Code

All criteria have to be checked for well defined loading conditions. In the calculations for the evaluation of the **general intact stability criteria** (1a – 4 in Figure 6) the GZ-curve is referring to calm water conditions: no waves are considered.

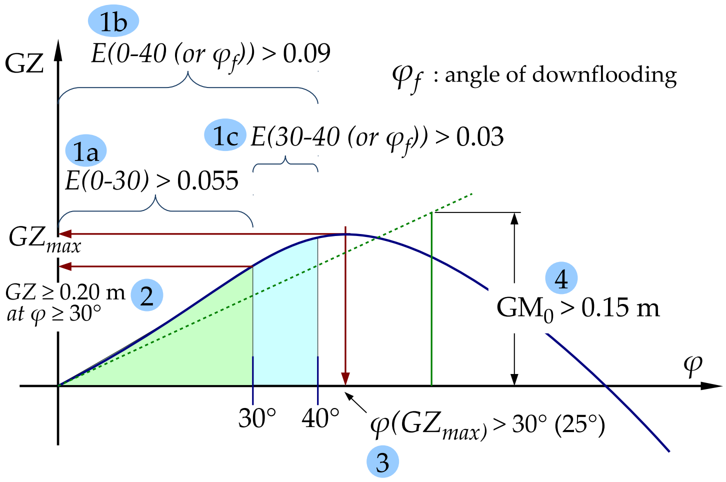


Figure 6: IMO IS Code: General intact stability criteria for all ships

The ability of a vessel to withstand the combined effects of beam wind and rolling is to be demonstrated: the **weather criterion** which however also refers to the calm water GZ-curve, see Figure 7. The underlying assumption: the ship rolls from angle φ_0 to φ_1 to φ_2: $A_2 \geq A_1$.

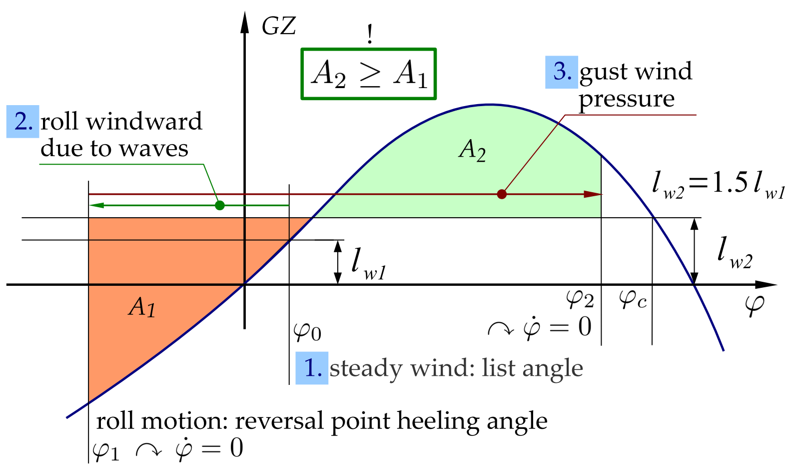


Figure 7: IMO IS Code: Severe wind and rolling criterion: weather criterion

10 Hull Form Properties

In the following, the index "S" refers to properties calculated based on mould line data, properties without an index include the approximated shell plating.

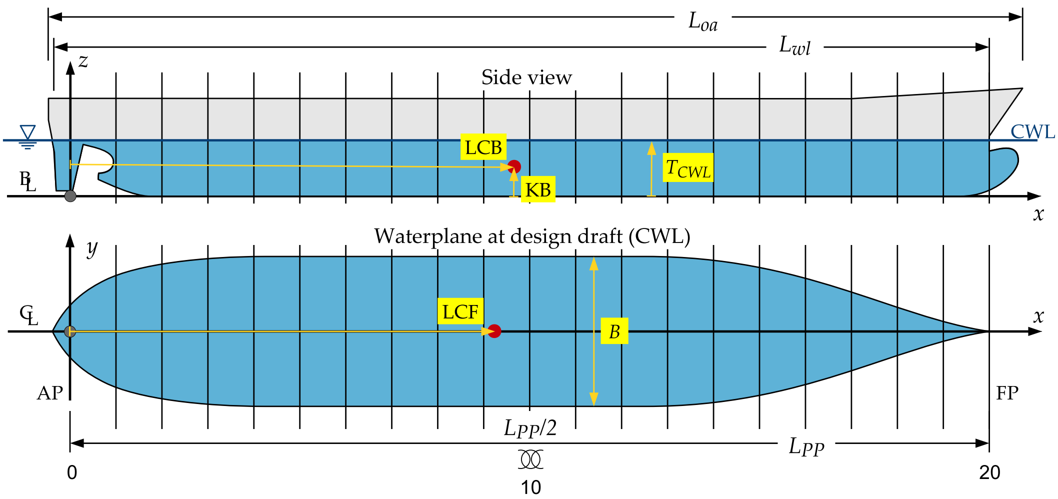


Figure 8: Coordinate system to calculate hull form properties

A_X; A_M $[m^2]$ ↷ Area of hull cross section at any $x = const$; midship hull cross section

$$A_X(T) = 2\int_0^T y_S(T)\,dz \quad ; \quad A_M(T) = A_X(T)\big|_{x=L_{pp}/2}$$

C_X; C_M [-] ↷ Section coefficient at $A_X(T)_{max}$; midship section coefficient

$$C_X(T) = \frac{A_X(T)_{max}}{B(T)\cdot T} \quad ; \quad C_M(T) = \frac{A_M(T)}{B(T)\cdot T}$$

A_w $[m^2]$ ↷ Water plane area at any $z = T = const$

$$A_{W_S}(T) = 2\int_l y_S(x,T)\,dx \quad ; \quad A_W(T) \approx 1.004\cdot A_{W_S}(T)$$

C_{WP} [-] ↷ Water plane area coefficient at any $z = T = const$

$$C_{WP}(T) = \frac{A_{W_S}(T)}{L_{wl}(T)\cdot B(T)}$$

LCF, X_W $[m]$ ↷ Longitudinal centre of floatation

$$LCF_S(T) = X_{W_S}(T) = \frac{2\int_l x\cdot y_S(x,T)\,dx}{A_{W_S}} \quad ; \quad LCF(T) \approx LCF_S(T)$$

∇ $[m^3]$ ↷ Displaced volume

$$\nabla_S(T) = 2\int_l\int_0^T y_S(x,z)\,dzdx = \int_l A_S(x,T)\,dx = \int_0^T A_{W_S}(z)\,dz \quad ; \quad \nabla_{CWL} \approx 1.005\cdot\nabla_{S_{CWL}}$$

C_B [-] $\curvearrowright$ Block coefficient

$$C_B(T) = \frac{\nabla_S(T)}{L_{pp} \cdot B(T) \cdot T}$$

C_P [-] $\curvearrowright$ Prismatic coefficient

$$C_P(T) = \frac{\nabla_S(T)}{A_X(T)_{max} \cdot L_{pp}} = \frac{c_B(T)}{c_X(T)}$$

Δ [t] $\curvearrowright$ Displacement

$$\Delta(T) = \rho_{sea} \cdot \nabla(T) \quad ; \quad \Delta(T) \approx 1.025 \cdot \nabla(T) \quad ; \quad \Delta(T) \approx 1.03 \cdot \nabla_S(T)$$

LCB [m] $\curvearrowright$ Longitudinal centre of buoyancy

$$LCB_S(T) = \frac{\int_l x \cdot A_S(x,T)\, dx}{\nabla_S(T)} = \frac{\int_0^T x_W(z) \cdot A_{W_S}(z)\, dz}{\nabla_S(T)} \quad ; \quad LCB \approx LCB_S(T)$$

KB [m] $\curvearrowright$ Vertical centre of the displaced volume

$$KB_S(T) = \frac{\int_0^T z \cdot A_{W_S}(z)\, dz}{\nabla_S(T)} \quad ; \quad KB(T) \approx 0.998 \cdot KB_S(T)$$

KM [m] $\curvearrowright$ Metacentre

$$KM_S(T) = BM(T) + KB_S(T) = \frac{I_{y_S}(T)}{\nabla_S(T)} + KB_S(T) = \frac{\frac{2}{3}\int_l y^3(x,T)\, dx}{\nabla_S(T)} + KB_S(T)$$

$$KM(T) \approx \frac{I_{y_S}(T)}{\nabla(T)} \left(1 + \frac{2\nabla_{A_{CWL}}}{\nabla_{CWL}}\right) + KB(T) \approx 1.012 \cdot \frac{I_{y_S}(T)}{\nabla(T)} + KB(T)$$

KM_L [m] $\curvearrowright$ Longitudinal Metacentre

$$KM_{L_S}(T) = BM_L(T) + KB_S(T) = \frac{I_x(T) - x_{W_S}^2(T) \cdot A_{W_S}(T)}{\nabla_S(T)} + KB_S(T)$$

$$= \frac{2\int_l x^2 y(x,T)\, dx - x_{W_S}^2(T) \cdot A_{W_S}(T)}{\nabla_S(T)} + KB_S(T)$$

$$KM_L(T) \approx 1.004 \cdot \frac{I_{x_S}(T)}{\nabla(T)} + KB(T)$$

E_T [t] $\curvearrowright$ Moment to alter trim one unit

$$E_{T_S}(T) = \frac{\rho \cdot I_{x_S}(T)}{L_A} \quad ; \quad E_T(T) \approx 1.004 \cdot E_{T_S}(T)$$

List of Symbols and Acronyms

For SI-units holds:

Dimension symbol	Unit symbol
L	m
M	kg
T	s

∇	L^3	displaced volume
∇_A	L^3	displaced volume of shell plating
$\nabla_{A_{CWL}}$	L^3	displaced volume of shell plating at design draft
∇_{CWL}	L^3	displaced volume at design draft
∇_F	L^3	displaced volume in fresh water
∇_S	L^3	displaced volume on moulded lines; displaced volume in sea water
γ	$M/(L^2T^2)$	specific weight (unit weight) ($\gamma = \rho \cdot g$)
Δ	M	displacement ($\Delta = \nabla \cdot \rho$)
ΔGM	L	change of metacentric height
$\delta\varphi$	$-$	small rotation about ξ or x-axis resulting in heel
$\delta\psi$	$-$	small rotation about η or y-axis resulting in trim
δR_x	ML/T^2	small change of moment about principle transverse axis
δR_y	ML^2/T^2	small change of moment about principle longitudinal axis
δR_z	ML^2/T^2	small change of force in vertical direction
δT	L	small translation in vertical direction: change of draft
ζ	L	coordinate value in direction of ζ-axis, here for ships upwards
ζ_B	L	centroid of buoyancy in direction of ζ-axis
ζ_G	L	centroid of gravity in direction of ζ-axis
η, ξ, ζ	$-$	coordinate system related to the fluid surface, η, ξ define plane parallel to fluid surface, ζ orthogonal to fluid surface
η	L	coordinate value in direction of η-axis, here for ships to portside
η_B	L	centroid of buoyancy in direction of η-axis
η_G	L	centroid of gravity in direction of η-axis
η_w	L	centroid of water plane in direction of η-axis
λ	$-$	expresses the effect of a hull form not being wall sided
ξ	L	coordinate value in direction of ξ-axis, here for ships in longitudinal direction
ξ_B	L	centroid of buoyancy in direction of ξ-axis
ξ_G	L	centroid of gravity in direction of ξ-axis
ξ_w	L	centroid of water plane in direction of ξ-axis
ρ	M/L^3	density of fluid, not further specified
ρ_{air}	M/L^3	density of air ($\sim 1.25\ kg/m^3$)
ρ_F	M/L^3	density of fresh water, in naval architectural calculations taken as 1000 kg/m^3
ρ_S	M/L^3	density of sea water, in naval architectural calculations taken as 1025 kg/m^3
ρ_T	M/L^3	density of fluid in tank, not further specified
ρ_W	M/L^3	density of water, not further specified
φ	$-$	angle of rotation about x-axis (ξ-axis) – heel
φ_{dyn}	$-$	dynamic heeling angle
φ_{list}	$-$	angle of list
φ_{stat}	$-$	static heeling angle
$\varphi_{stat_{max}}$	$-$	maximum static heeling angle
φ_f	$-$	angle of downflooding
$\dot{\varphi}$	$1/T$	roll velocity

$\ddot{\varphi}$	$1/T^2$	roll acceleration
ψ	$-$	angle of rotation about y-axis (η-axis) – trim
6-DOF		six degrees of freedom: 3 translations and 3 rotations
A	L^2	area
A_M	L^2	area of immersed hull cross section at midship position
A_W	L^2	water plane including approximated shell plating
A_{W_S}	L^2	water plane on mould lines
A_X,	L^2	area of immersed hull cross section at position x
B	ML/T^2	buoyancy force
B	L	ship moulded breadth
B	$-$	symbol for centroid of buoyancy
℄B (BL)		base line
BM	L	vertical distance of M from B, (transverse) metacentric radius
BM_L	L	vertical distance of M from B, longitudinal metacentric radius
℄		centre line
CWL	$-$	water line at design draft
c_B	$-$	block coefficient
c_X (c_M)	$-$	(midship) section area coefficient
c_P	$-$	prismatic coefficient
c_{WP}	$-$	water plane coefficient
D	L	ship moulded depth
DOF		degree of freedom to translate or rotate an object
E_{30}	$L\,rad$	area under righting arm curve in range of $0° \leq \varphi \leq 30°$
E_{40}	$L\,rad$	area under righting arm curve in range of $0° \leq \varphi \leq 40°$
E_{30-40}	$L\,rad$	area under righting arm curve in range of $30° \leq \varphi \leq 40°$
E_T	M	moment to alter trim by one meter
G	ML/T^2	gravity
G	$-$	symbol for centroid of mass distribution
GM	L	vertical distance between G and M, (transverse) metacentric height ($GM = KB + BM - KG = KM - KG = \partial GZ/\partial\varphi(\varphi = 0)$)
GM_R	L	reduced metacentric height: including free surface correction
GM_0	L	initial metacentric height
GM_L	L	vertical distance between G and M_L, longitudinal metacentric height ($GM_L = KB + BM_L - KG = KM_L - KG$)
GZ	L	righting arm ($\rightarrow Z$)
GZ_{corr}	L	GM correction due to partially filled tanks
GZ_{cw}	L	righting arm curve in clam water
GZ_{crest}	L	righting arm curve for wave crest at midships
GZ_{max}	L	maximum righting arm
GZ_{trough}	L	righting arm curve for wave trough at midships
g	L/T^2	gravitational acceleration constant ($9.81 m/s^2$)
I_η	L^4	water plane moment of inertia about ξ-axis
I_{η_S}	L^4	water plane moment of inertia about the principle ξ-axis
I_ξ	L^4	water plane moment of inertia about η-axis
I_{ξ_S}	L^4	water plane moment of inertia about the principle η-axis
$I_{\xi\eta}$	L^4	water plane product moment of area, in ξ - η coordinate-system
$I_{\xi\eta_S}$	L^4	water plane product moment of area, about principle ξ - η co-system
I_{Fys}	L^4	moment of inertia of free fluid surface in transverse direction
I_x	L^4	water plane moment of inertia about y-axis
I_{x_S}	L^4	water plane moment of inertia about the principle y-axis
I_{xy}	L^4	product moment of water plane,

I_{xy_S}	L^4	product moment of water plane in principle X-Y co-system
I_y	L^4	water plane moment of inertia about x-axis
I_{y_S}	L^4	water plane moment of inertia about the principle x-axis in x-y coordinate-system
I_L	L^4	water plane longitudinal moment of inertia
I_T	L^4	water plane transverse moment of inertia
IMO		International Maritime Organization
IS-Code		Intact Stability Code of the International Maritime Organization
K	–	"Keel point": the intersection of centre line ℄ and base line BL in a 2D section view
KB	L	vertical distance of B from K: position of vertical centroid of buoyancy
KG	L	vertical distance of G from K: position of vertical centroid of mass
KM	L	vertical distance of M from K: (transverse) metacentre
KM_L	L	vertical distance of M_L from K: longitudinal metacentre
$KN(\varphi)$	L	cross curve of stability ($= \eta_B(\varphi)$)
k	L	heeling arm (M_k/B)
L	L	ship length, not further specified
L_A	L	distance between aft and forward draft marks, measured in the ship longitudinal direction
L_{pp}	L	ship length between perpendiculars
L_{oa}	L	ship length over all
L_{WL}	L	ship waterline length ($= f(T)$)
LCB	L	longitudinal centroid of buoyancy in longitudinal direction
LCF	L	longitudinal centre of floatation, centroid of water plane in longitudinal direction
LCG	L	longitudinal centre of gravity, centroid of mass in longitudinal direction
LLC66/88		International **L**oad **L**ine **C**onvention, IMO Safety Regulation
$l_{w\{1,2\}}$	L	wind heeling lever, defined in the IS-Code weather criterion
M	–	(transverse) metacentre (location on centre line)
M_{∇_ζ}	L^4	vertical moment of displaced volume ($M_{\nabla_\zeta} = \nabla \cdot \zeta_B$)
$M_{B_{\{x,y\}}}$	ML^2/T^2	moment of buoyancy force about y-axis, about x-axis
$M_{B_{\{\xi,\eta\}}}$	ML^2/T^2	moment of buoyancy force area about η-axis, about ξ-axis
$M_{G_{\{x,y\}}}$	ML^2/T^2	moment of gravity force about y-axis, about x-axis
$M_{G_{\{\xi,\eta\}}}$	ML^2/T^2	moment of gravity force area about η-axis, about ξ-axis
M_k	ML^2/T^2	upsetting, heeling moment
M_L	–	longitudinal metacentre (location on centre line)
M_P	ML^2/T^2	heeling moment due to crowding of people
M_{ST_n}	L^3	vertical moment of immersed part of a station n
M_T	ML^2/T^2	heeling moment due to turning *or* due to towing hawser
M_Y	ML^2/T^2	yawing moment
M_{W_0}	ML^2/T^2	heeling moment due to wind in upright position
$M_{w_{\{x,y\}}}$	L^3	moment of water plane about y-axis, about x-axis
$M_{w_{\{\xi,\eta\}}}$	L^3	moment of water plane about η-axis, about ξ-axis
m	M	mass
N	-	in a 2D section view: intersection of line through K parallel to the water surface and the action line of the buoyancy vector
PORT, PS		port side
R_η	ML^2/T^2	moment about principle longitudinal axis (ξ)
STB, STBD		starboard side
T	L	moulded draft

T_{AP}	L	moulded draft at aft perpendicular
T_{CWL}	L	moulded design draft
T_{SCL}	L	moulded draft for calculation of scantlings, larger than T_{CWL}
T_{FP}	L	moulded draft at forward perpendicular
TCB	L	transverse centre of buoyancy, centroid of buoyancy in transverse direction, equal to zero for symmetrically shaped ship hull forms floating without list
TCF	L	transverse centre of flotation, centroid of water plane area in transverse direction
TCG	L	transverse centre of gravity, centroid of mass in transverse direction, equal to zero for symmetrically designed ships
t	L	trim ($T_{FP} - T_{AP}$)
x	L	x-coordinate in (ship) longitudinal direction
x_B	L	centroid of buoyancy in longitudinal direction (LCB)
x_G	L	centroid of gravity in longitudinal direction (LCG)
x_W	L	centroid of water plane in longitudinal direction (LCF)
y	L	y-coordinate in (ship) transverse direction
y_B	L	centroid of buoyancy in transverse direction
y_G	L	centroid of gravity in transverse direction
y_W	L	centroid of water plane in transverse direction
y_{WL}	L	distance from centre plane to local breadth of water plane $(= f(x, T, t)$
Z	-	in a 2D station view: intersection of line through G parallel to the water surface and the action line of the buoyancy vector ($\rightarrow GZ$)
z	L	z-coordinate in (ship) vertical direction
z_∇	L	centroid of displaced volume in vertical direction
z_B	L	centroid of buoyancy in vertical direction
z_G	L	centroid of gravity in vertical direction
z_M	L	location of metacentre in vertical direction

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