– Ship Safety –
Damaged Stability
Roll Motions in Waves

IN A NUTSHELL

Compendium

Relevant to Questions in Exam

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1 Ship Safety Stability in Damaged Conditions

1 Ship Safety: General Aspects of Damages

The occurrence of a ship damage can be categorized according to its reason in e.g.:

- grounding,
- collision,
- fire, explosion,
- water ingress through openings,
- structural failure due to dynamic (sea) loads,
- … corrosion,
- … fatigue.

The reasons leading to the vessel's total loss (see Figure 1) due to

! loss of buoyancy and/or stability can be caused by a previous

! partial loss of structural integrity finally resulting in a hull girder collapse.

A rational approach to evaluate the ship safety is based on the assessment of the operational *risk*. Risk (R) can be expressed by the likelihood of the occurrence of an adverse event multiplied by the related consequences. The risk value should be as low as reasonably practicable: $R \leq ALARP$ which is defined in regulations to be observed.

The risk assessment can be performed based on different approaches which are to be distinguished in (formerly → today/tomorrow)

- Qualitative (past experience) → Quantitative (simulations approved by experiments)
- Deterministic (specific scenarios) → Probabilistic (all possible scenarios)

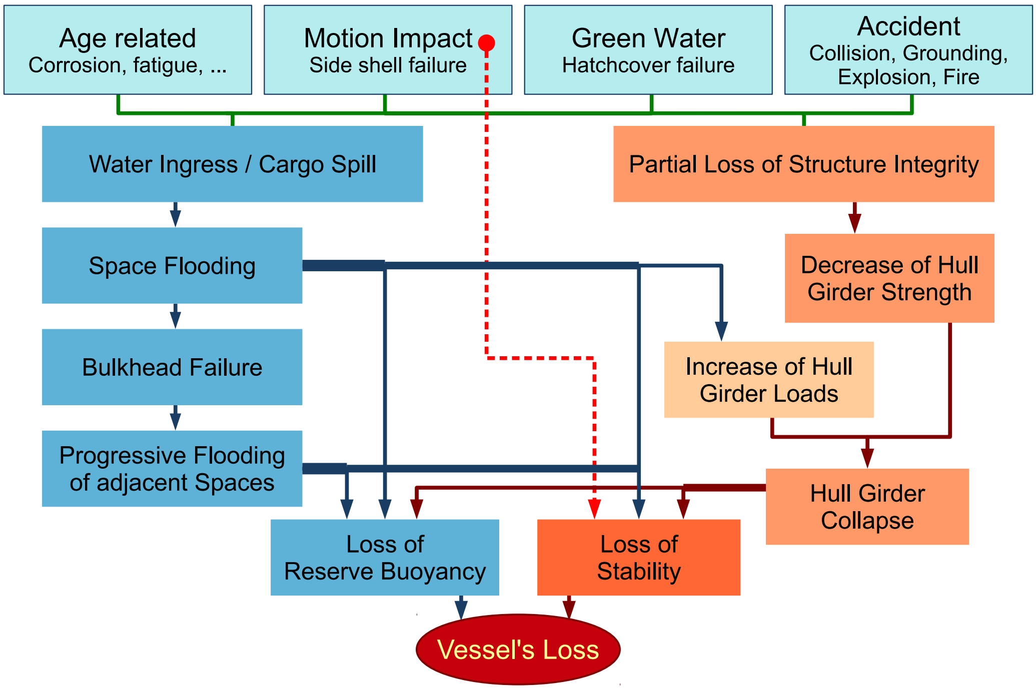

Figure 1: Damage scenarios yielding to vessel's loss. Red dotted line: direct impact of motions on stability, see Chapter 2 on roll motions in waves

In the following the focus will be on grounding and collision scenarios potentially leading to the vessel's loss as well as on extensive ship roll motions potentially leading to large accelerations and in the worst case in capsizing of the vessel in a severe seaway.

2 Stability in Damaged Conditions

Three principal types of damage according to the location and corresponding safety measures are to be distinguished, see Table 1.

Damage Type	Principal Location	Safety Related Measures
Collision	Side	Transverse and Longitudinal Bulkheads
Grounding	Bottom	Double Bottom, Bulkheads
Ramming	Stem	Collision Bulkhead

Table 1: Damage types vs. safety related measures

Two principally different concepts for the determination of the ship's safety in case of a damage are to be distinguished: **deterministic** versus **probabilistic** approach.

To evaluate the floating condition after a damage has occurred, the change in floating position (draft $\Delta T = T - T_0$, trim $\delta t = t - t_0$) and the remaining stability have to be analysed. Stability characteristics are given by ΔGM and the GZ curve taking into account a potentially considerable list and trim in intermediate as well as in the final floating position.

Definition of Permeability: Percentage of a volume/space (κ_v) or area/surface (κ_a) which can be occupied by water ($0 \leq \kappa_{\{a,v\}} \leq 1$).

2.1 Floating Position After Damage

Two different methods can be applied for calculating the floating position after a damage of a compartment or group of adjacent compartments.

Convention throughout the following: variables in capital letters represent ship properties, variables in small letters represent properties of the flooded compartment (group). Index 0: before damage, index T after damage in final floating position but object assumed to be intact, index R for "**r**emaining object" after damage in final floating position.

For a compartment located "at LCB": parallel sinkage ($\Delta T = T_0 \to T$) $\curvearrowright$ no trim: $\delta t = 0$:

1. Calculation Method **"Loss of Buoyancy"** or **"Constant Displacement"**

$$\Delta = const\,! \;\curvearrowright\; \nabla_R = \nabla_0; \quad KB = \uparrow; \quad KG = const; \quad LCB \neq const; \quad LCF \neq const$$

$$\int_{T_0}^{T} (A_{WL}(T) - a_{WL}(T))\, dT = \kappa_v \cdot v_0$$

2. Calculation Method **"Additional Weight"**

$$\Delta \neq const\,! \curvearrowright \nabla_T = \nabla_0 + \kappa_v \cdot v_T; \quad KB = \uparrow; \quad KG \neq const; \quad LCB \neq const; \quad LCF \neq const$$

$$\kappa_v \cdot v_T = \Delta V = \int_{T_0}^{T} A_{WL}(T)\, dT = \kappa_v \cdot \left(v_0 + \int_{T_0}^{T} a_{WL}(T)\, dT \right)$$

For a symmetrical (to center line) damage at any longitudinal position yielding sinkage and trim but no list:

$$\sum \text{forces} = 0 \quad \curvearrowright \quad \nabla_T - \kappa_v \cdot v_T = \nabla_0$$
$$\sum \text{moments} = 0 \quad \curvearrowright \quad \nabla_T \cdot a_1 = \kappa_v \cdot v_T \cdot a_2$$

2.2 Change (Loss) of Initial Stability After Damage (Loss of Buoyancy Method)

1. Symmetrical (to center line) damage at any longitudinal position: sinkage and trim:

$$\Delta GM = GM_0 - GM_R = \underbrace{BM_0 - BM_R}_{\Delta BM} - \underbrace{(KB_R - KB_0)}_{\Delta KB}$$

$$\Delta GM = \frac{\kappa_a \cdot i_T}{\nabla_0} - \frac{\Delta I_B}{\nabla_0} + \frac{\kappa_v \cdot v_T}{\nabla_0} \cdot (kb_T - KB_T) - (KB_T - KB_0)$$

Special case of symmetrical damage "at LCB": parallel sinkage but no trim:

$$\Delta GM \approx \frac{\kappa_a \cdot i_T}{\nabla_0} - \frac{\Delta I_B}{\nabla_0} + \frac{\kappa_v \cdot v_0}{\nabla_0} \cdot \left(T_0 + \frac{\Delta T}{2} - kb_0 \right)$$

2. Asymmetrical (to center line) damage at any longitudinal position ($\curvearrowright$ sinkage, trim and heel): change of coordinate axes for resulting water plane (without flooded compartment):

1. translation: $x_0 + \Delta x, y_0 + \Delta y \to x, y$: area centroid $CF_T \to CF_R$
2. *and* rotation (α): $x, y \to x', y'$ yielding the **relevant principle coordinate axes** ($I_{x'y'} = 0$) with $I_{x'} = I_{B_{min}}$ and $I_{y'} = I_{L_{max}}$ about which the vessel heels and trims after damage.

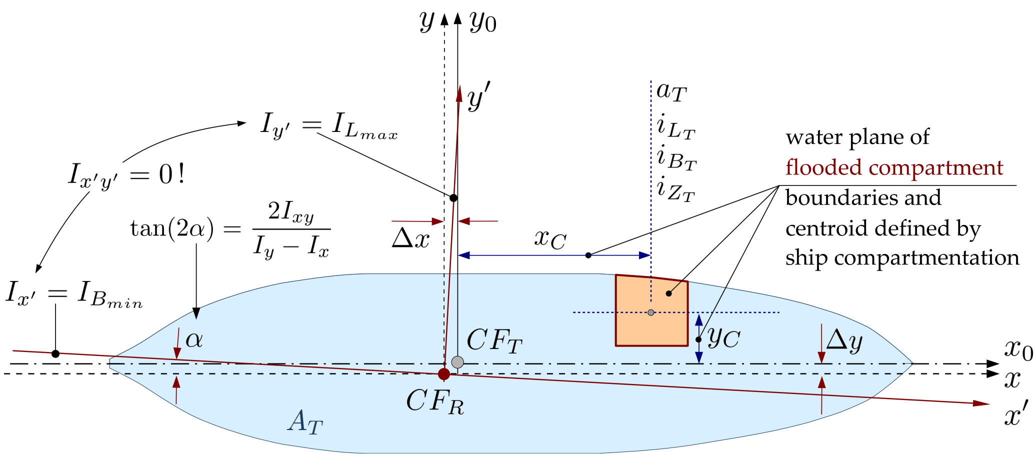

Figure 2: Asymmetrical damage to center line: coordinate systems in water plane view

$$\Delta GM = \Delta BM - \Delta KB = \frac{I_0 - I_{B_{min}}}{\nabla_0} - \Delta KB$$

$$I_{B_{min}} = \frac{I_{L_R} + I_{B_R}}{2} - \frac{I_{L_R} - I_{B_R}}{2} \cdot \sqrt{1 + \left(\frac{2I_{z_R}}{I_{L_R} - I_{B_R}}\right)^2}$$

$$I_{L_R} = I_y = I_{L_T} - (\kappa_a i_{L_T} + \kappa_a a_T \cdot x_C^2) - (A_T - \kappa_a a_T) \cdot \Delta x^2 \quad ; \quad \Delta x = \frac{\kappa_a a_T \cdot x_C}{A_T - \kappa_a a_T}$$

$$I_{B_R} = I_x = I_{B_T} - (\kappa_a i_{B_T} + \kappa_a a_T \cdot y_C^2) - (A_T - \kappa_a a_T) \cdot \Delta y^2 \quad ; \quad \Delta y = \frac{\kappa_a a_T \cdot y_C}{A_T - \kappa_a f_T}$$

$$I_{Z_R} = I_{xy} = I_{Z_T} - (\kappa_a i_{Z_T} + \kappa_a f_T \cdot x_C y_C) - (A_T - \kappa_a a_T) \cdot \Delta x \cdot \Delta y; \quad \tan(2\alpha) = 2I_{xy}/(I_y - I_z)$$

2.3 Evaluation of Damages: Deterministic Approach

Floodable length curve: maximum distance of transverse watertight subdivision as function of ship longitudinal position (x) observing specified criteria. Traditionally these are *minimum freeboard* ($a(x)$) and *minimum stability* ($b(x)$) of vessel after damage, see Figure 3. Calculation input: ship hull form, loading condition (KG, initial draft), FB_{min} and area permeability $\kappa_a = f(x)$ as well as volumetric permeability $\kappa_v = f(x)$.

Factor of subdivision ($F < 1$) intended to increase ship safety in case of damage: multiplier to local *maximum floodable* length resulting in *local permissible* length.

N-Compartment status: number N of adjacent compartments floodable while observing floating and stability criteria for damaged vessel.

Special case symmetrical damage “at LCB” ($\curvearrowright$ parallel sinkage (ΔT), no trim): maximum floodable length (a) according to freeboard criterion

$$\frac{a}{L} \approx \frac{C_{WP}}{\kappa_v} \cdot \frac{D - FB_{min} - T_0}{D - FB_{min} - T_0(1 - C_M)}$$

Fundamental arguments against the deterministic approach are: 1) Future damages are de-

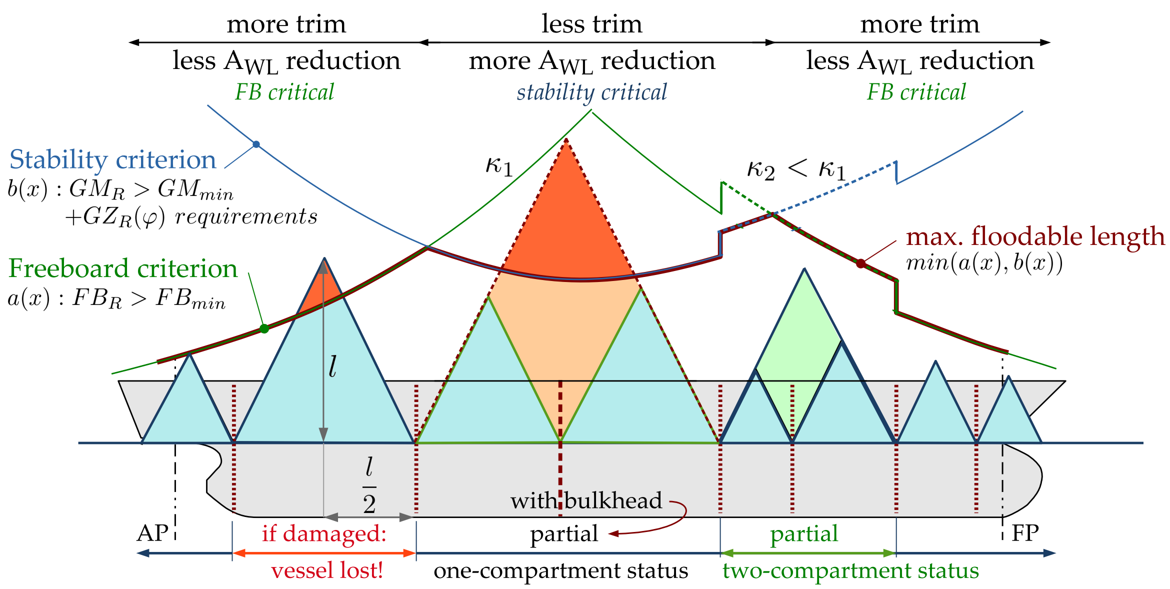

Figure 3: Principle sketch of "floodable length curve"

termined and not considered as a random phenomenon. 2) Longitudinal and horizontal watertight compartmentation is not considered. 3) Factor of subdivision can result in reduced safety! 4) N-compartment status pretends increased safety: but e.g. a (small) damage at a bulkhead may result in vessel's loss even if one-compartment status realized for whole ship.

2.4 Evaluation of Damages: Probabilistic Approach

The underlying concept is rationally based compared to the deterministic approach:

1. Consider **damages** as a **random phenomenon**!
2. Determine all compartment (groups) of the specific ship design (hull compartmentation, loading condition) which if damaged will result in an acceptable floating position and stability characteristics $\curvearrowright$ safe compartments (groups). Note that different criteria can be applied to precisely formulate the acceptance of a floating position, see section 2.10.2.
3. For each safe compartment (group) "i" determine the probability that only that compartment (group) will be opened in an accident $\curvearrowright$ partial survivability Δp_i.
4. Sum up all partial survivabilities Δp_i which results in the total vessel's survivability:
$$P = \sum_{i=1}^{n} \Delta p_i < 1$$
5. Assess the vessel's survivability P with respect to formulated minimum requirements e.g. legally binding by IMO SOLAS – International Convention for the Safety of Life at Sea.

2.5 Damage Dimensions

A damage can be characterized by its bounding box and location with respect to the vessel's global coordinate system:

1. location in ship's longitudinal direction (x, dimensionless: $\xi = x/L$); for side damages measured at half damage length, for bottom damages measured at foremost extension,
2. damage length (l) measured in ship's longitudinal direction, with η the dimensionless damage length l/L,
3. penetration depth (t) measured in the ship's transverse direction (dimensionless: $\tau = t/B$),
4. penetration height (h) measured in the ship's vertical direction (dimensionless: $\zeta = h/D$).

2.6 Damage Statistics

The following statements can be derived from the damage statistic maintained by IMO:

1. Small damages (characterised by length) occur more often than larger ones.
2. The damage location for side damages is almost evenly distributed over the ship length, in the forward region however more pronounced.
3. The mean length value (not the average value!) for side damages is $\eta_{50} \approx 0.0555$.
4. Bottom damages occur far more often in the forward region than in the aft region.
5. The mean length value for bottom damages is $\eta_{50} \approx 0.103 \curvearrowright$ approximately two times larger than for side damages (transformation of the vessel's energy due to $v > 0$)!
6. The damage penetration depth (either transversely for side damages or vertically for bottom damages) is to be treated separately as the statistical data do not allow to derive the corresponding three parameter probability density functions. Therefore $p(\xi, \eta, \tau)$ is to be calculated $p(\xi, \eta) \cdot p_\tau(\eta)$, likewise: $p(\xi, \eta, \zeta) = p(\xi, \eta) \cdot p_\zeta(\eta)$.
7. For stem damages (due to ramming) the penetration mean value measured from stem backwards can be taken as $\approx 0.05L$.

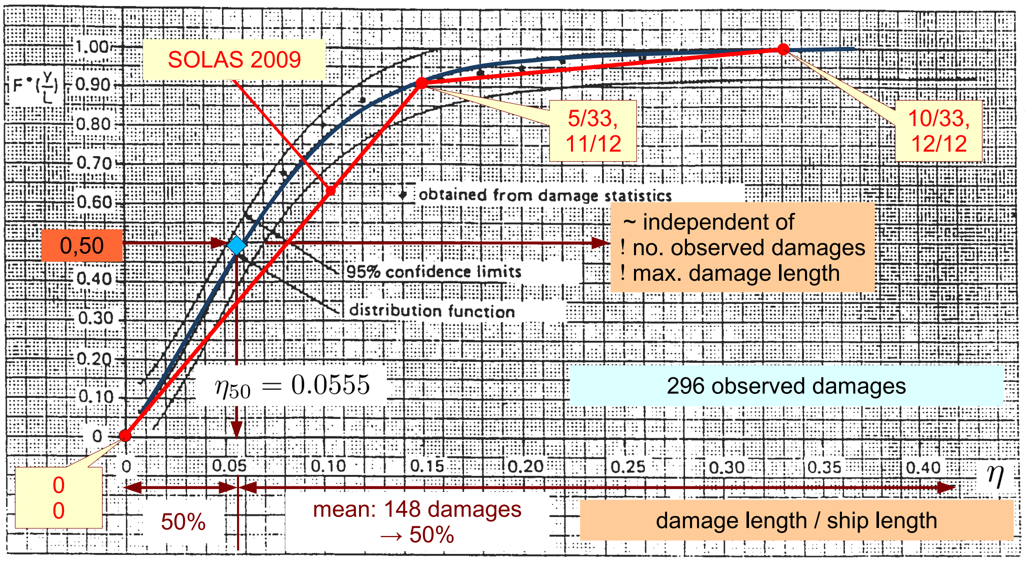

Figure 4: Statistic: side damage length – distribution function

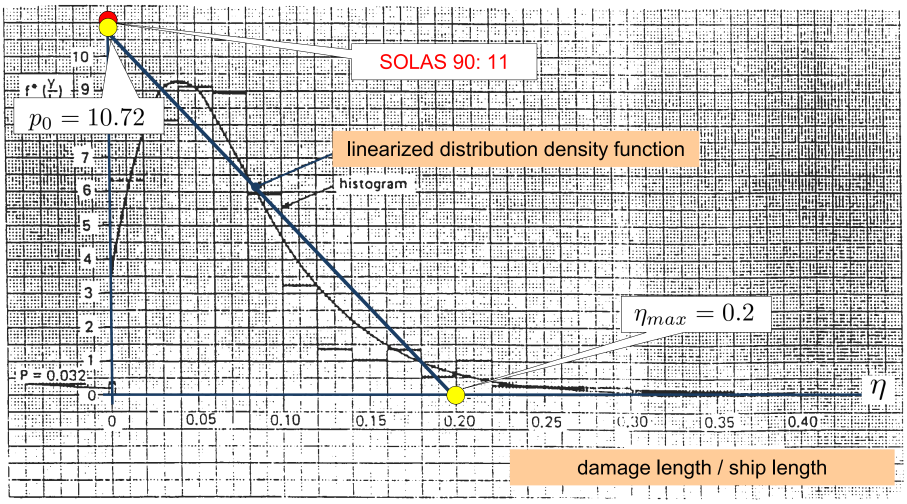

Figure 5: Statistic: side damage length – linearized distribution density function

2.7 Side Damage: Calculation Approach for Partial Survivability

The approach to calculate the partial survivability for safe compartments (groups):

1. Derive the probability density function based on the damage statistic: e.g. for side damages the assumptions might hold: a) damages are evenly distributed over the ship length, b) the damage length distribution density function can be approximated by a linear function: $p(\xi, \eta) \curvearrowright p(\eta) = 10.72(1 - 5\eta)$ with $\eta_{max} = 0.20$, see Figure 5.
2. Calculate the integral of the distribution density function over the area of possible damages which yields the partial survivability function Δp, see Figure 6:

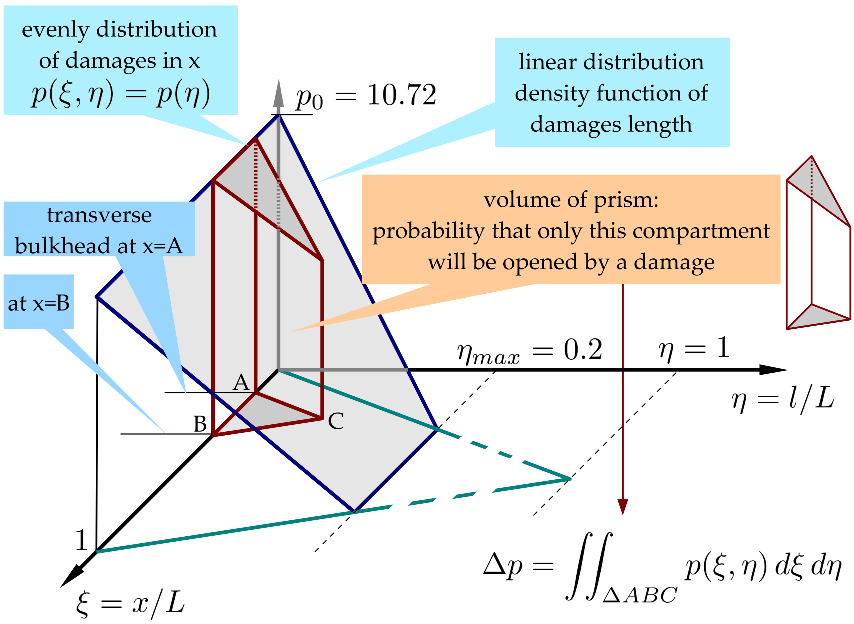

Figure 6: Probability opening a single compartment between A and B

$$\Delta p\Big|_{l/L<\eta_{max}} = \int_0^{l/L}\int_{\eta/2}^{l/L-\eta/2} 10.72(1-5\eta)\, d\xi\, d\eta = 5.36\left(\frac{l}{L}\right)^2 - 8.93\left(\frac{l}{L}\right)^3$$

$$\Delta p\Big|_{l/L>\eta_{max}} = \int_0^{0.2}\int_{\eta/2}^{l/L-\eta/2} 10.72(1-5\eta)\, d\xi\, d\eta = 1.072\left(\frac{l}{L}\right) - 0.072$$

3. In case of no longitudinal bulkheads exist: the partial survivability Δp of a safe compartment (group) can simply be calculated with the corresponding compartment (group) length $l = x_B - x_A$, the distance between the bounding transverse watertight bulkheads.

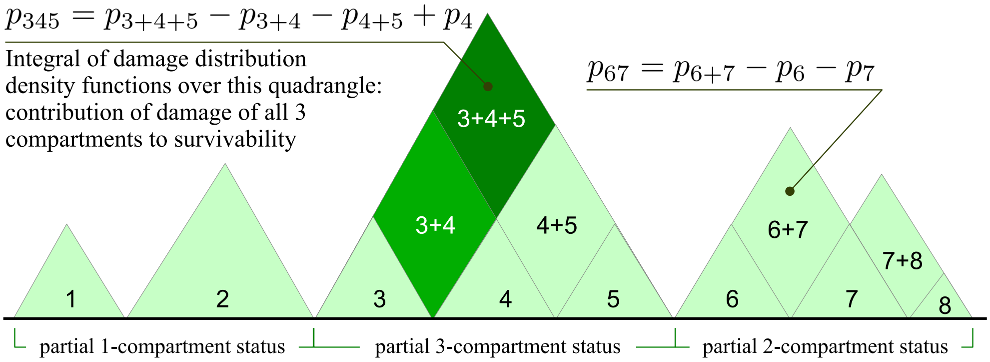

Figure 7: Contribution of an exemplary compartment configuration to survivability

4. In case of a longitudinal bulkhead exists between transverse bulkheads the partial survivability according to the above formula has to be corrected depending on the distance between half breadth and the longitudinal bulkhead (b).

$$\Delta p\Big|_{\tau_{max}=b/B} = \sum_{j=1}^{m} \partial\Delta p(\eta_j)\cdot p_{\tau_{max}}(\eta_j)$$

With $\partial\Delta p(\eta_j)\cdot p_{\tau_{max}}(\eta_j)$ expresses the probability that the compartment is opened by a side damage of length $\eta_{j1} < \eta_j < \eta_{j2}$ and the penetration depth is less than the distance between half breadth and the longitudinal bulkhead: $\tau_{max} = b/B$. See Figure 8 for the dependency of Δp on τ with an example $\eta = 0.09$ and $b = 0.15\cdot B$ which yields the partial survivability of that compartment $\Delta p \approx 2.3\%$.

2.8 Bottom Damage: Calculation Approach for Partial Survivability

Same approach as for side damages but damage statistic in this case yields the density function $p = f(\xi,\eta) = 12\xi - 24\eta$! Therefore the partial survivability Δp strongly depends on both, the damage length l/L and the damage location ξ_L!

$$\Delta p(\xi_L, l/L) = \iint p(\xi,\eta)\, d\xi\, d\eta$$

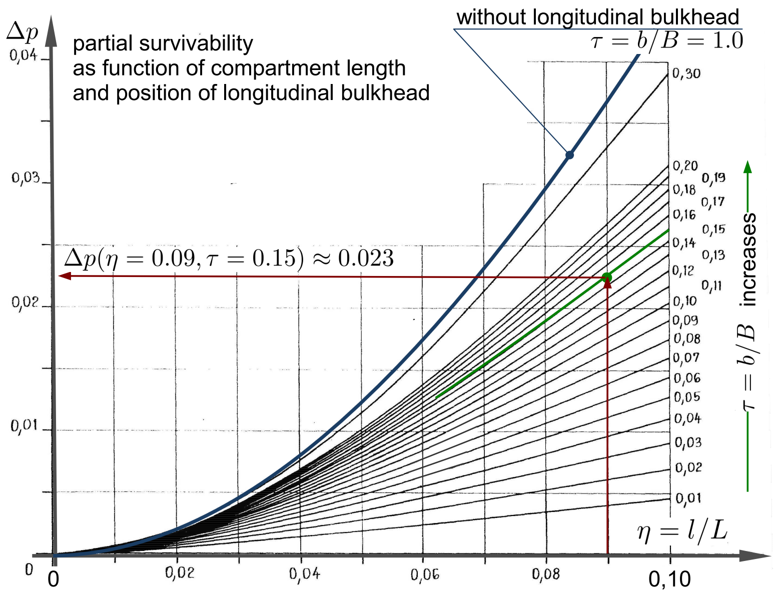

Figure 8: Side damage: partial survivability with longitudinal bulkhead at b/B

Figure 9 shows the result of this double integral for possible parameter damage position ξ_l (curve parameter) and compartment length l/L. The orange curve: according to the definition of the damage position, the damage length can never exceed the distance from the foremost damage extension to AP. Note the high dependency of the partial survivability on the damage location: a safe compartment of 40% length ($\eta = 0.40$) contributes from almost 60% to the vessel's survivability if located at FP to as low as ~6.5% if located in aftmost possible region.

Figure 10 shows the strong impact of a vertical compartment boundary (deck) on the partial

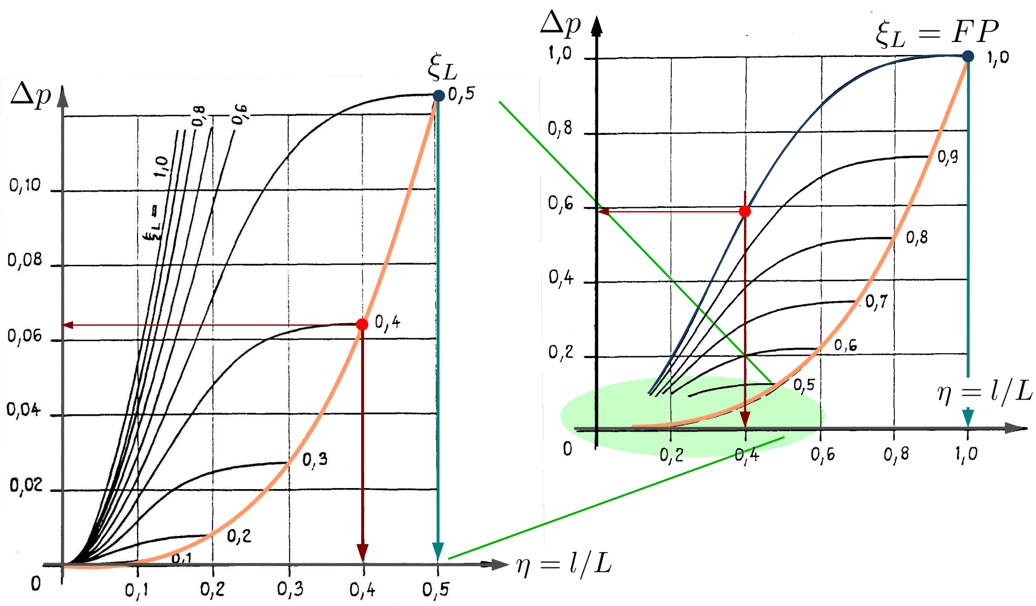

Figure 9: Bottom damage: partial survivability, no inner bottom $\Delta p(\xi_L, \eta)$

survivability for a compartment in case of a bottom damage: $\Delta p(\xi_L, \eta, \zeta)$. Three exemplary damage locations (graphs) are depicted: $\xi_L = \{1.0, 0.5, 0.3\}$. Note that the ordinate axes are not drawn in the same scale! The curve parameter is the normalized height of the watertight deck representing the vertical extend of the compartment (group) under concern.

Again the dependency of the damage location can be recognized: the partial survivability varies between ∼17% if the compartment (group) is located at FP to as low as ∼1.5% if it is located in the aftmost possible region. The partial survivability of a compartment aft of $L_{pp}/2$, length 30% ship length varies between ∼9% for a deck at 50% depth and ∼4.5% for a deck at 10% depth.

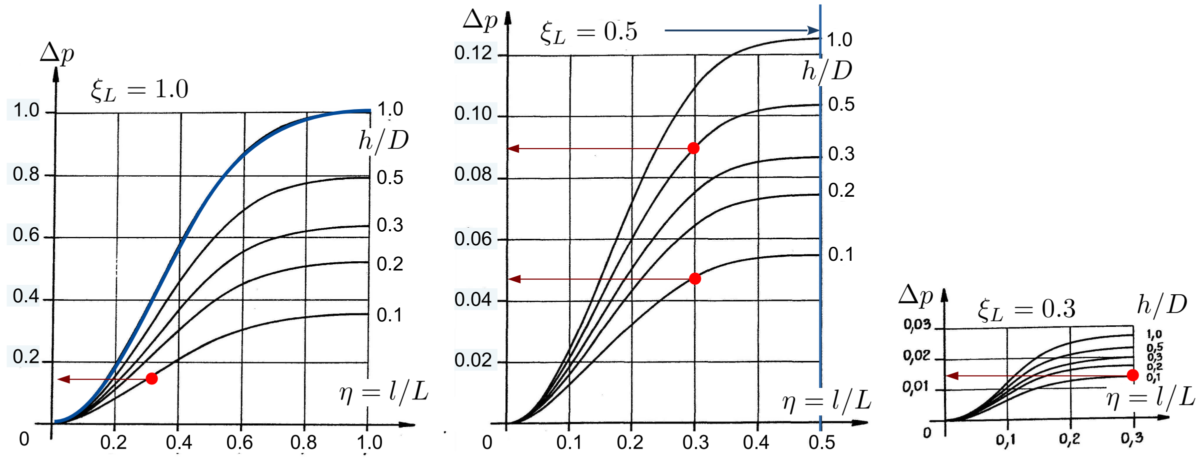

Figure 10: Bottom damage: partial survivability in existence of watertight deck $\Delta p(\xi_L, \eta, \zeta)$

2.9 Ramming – Stem Damage: Calculation Approach for Partial Survivability

Under the assumption of a linear distribution of the damage length measured from FP backwards in case of ramming and a mean length value of $r_{50} = 0.05L$ the probability density function becomes

$$p(r) = 11.716L^{-1} - 68.629L^{-2} \cdot r \quad \text{for} \quad 0 < r < 0.1707L$$

The probability of a damage with a length "a" therefore can be calculated to

$$P = \int_0^a p(r)\,dr = 11.716L^{-1} \cdot a - 34.315L^{-2} \cdot a^2 \quad \text{for} \quad a < 0.1707L$$

The minimum distance of the collision bulkhead rcb from FP for a given survivability P of the forepeak becomes:

$$rcb = 0.1707 - \sqrt{0.02914 - \frac{P}{34.315}}$$

2.10 IMO SOLAS Convention – From Physics to Regulations

The IMO "International Convention for the Safety of Life at Sea" (SOLAS) approach to define a minimum safety level of a vessel in case of a damage is based on a *required index R* which should be less or equal the ship specific *attained index A*: $\mathbf{A} \geq \mathbf{R}$!.

2.10.1 Required Index

The required index R is a function of the vessel size (specially defined ship length L_S). For passenger vessels the percentage of the lifeboat capacity with respect to the total number of persons on board is also considered.

1. For cargo ships with $L_S > 100m$ (for length $80\,m \leq L_S \leq 100\,m$ to be interpolated):

$$R = 1 - \frac{128}{L_S + 152}$$

2. For passenger ships with $N = N_1 + 2N_2$ and N_1 number of persons for whom lifeboats are provided, N_2 number of persons the ship is permitted to carry in excess of N_1:

$$R = 1 - \frac{5000}{L_S + 2.5N + 15225}$$

2.10.2 Attained Index

The attained index A is to be calculated for three draughts (deepest subdivision draught A_s, partial subdivision draught A_p and light service draught A_l) with an operation profile of

$$A = 0.4 \cdot A_s + 0.4 \cdot A_p + 0.2 \cdot A_l$$

For each draught the corresponding index is to be calculated summing up all compartments (groups) according to the formula

$$A_{\{s,p,l\}} = \sum_{i=1}^{n} p_i \cdot s_i$$

i the index represents the compartment or group of compartments under consideration,

p_i accounts for the probability that only the compartment or group of compartments under consideration may be flooded taking any longitudinal subdivision into account but disregarding any horizontal subdivision. The factor p_i is principally calculated as discussed in section 2.7.

s_i accounts for the probability of survival after flooding the compartment or group of compartments under consideration and includes the effect of any horizontal subdivision

$$s_i = \min(s_{\text{intermediate,i}} \text{ or } s_{\text{final,i}} \cdot s_{\text{mom,i}})$$

Note the important difference to the simplified approach described in section 2.4 in which s_i was taken to the constant value "1" for all "save" compartment(groups). However in the

SOALS implementation, the factor s_i allows a more rational judgement about the safety in case of a damage as the resulting floating positions are analysed in a more comprehensive approach.

$s_{\textbf{intermediate,i}}$ is the probability to survive all intermediate flooding stages until the final equilibrium stage. The factor $s_{\text{intermediate,i}}$ is applicable only to passenger ships (for cargo ships = 1) and shall be taken as the least of the s-factors obtained from all flooding stages including the stage before equalization. Its actual value is a function of GZ_{max} and the range of stability after damage. To be taken as 0, if the intermediate heel angle exceeds 15°.

$s_{\textbf{final,i}}$ is the probability to survive in the final equilibrium stage of flooding. Its value is a function of GZ_{max}, the range of stability after damage and the equilibrium heel angle ϕ_e. To be taken as 0, if $\phi_e \geq 15°$ for passenger ships and $\phi_e \geq 30°$ for cargo ships.

When determining the positive righting lever (GZ) of the residual stability curve, the displacement used should be that of the intact condition. That is, the constant displacement method of calculation should be used.

$s_{\textbf{mom,i}}$ is the probability to survive additional heeling moments in the final floating position of passenger ships (to be taken as unity for cargo ships). Its value is a function of GZ_{max}, the intact displacement at the subdivision draught and a heeling moment M_{heel} which is to be calculated as follows:

$$M_{\text{heel}} = \max(M_{\text{Passenger}}, M_{\text{Wind}}, M_{\text{Survivalcraft}})$$

$M_{\textbf{Passenger}}$ Moment due to passenger crowding: $M_{\text{Passenger}} = (0.075 \cdot N_P) \cdot (0.45 \cdot B)$ with N_P the maximum number of passengers, B ship breadth,

$M_{\textbf{Wind}}$ Moment due to beam wind: $M_{\text{Wind}} = (P \cdot A \cdot Z)/9.806$ with P the wind pressure to be taken as $120N/m^2$, A the projected lateral area above water line, Z the distance of centroid of A to $T/2$,

$M_{\textbf{Survivalcraft}}$ Moment due to launching the fully loaded survival crafts to be calculated according to the actual configuration.

Horizontal watertight boundaries Where horizontal watertight boundaries are fitted above the waterline under consideration the s-value calculated for the lower compartment or group of compartments shall be obtained by multiplying the value by the reduction factor v_m, which represents the probability that the spaces above the horizontal subdivision will not be flooded.

2.10.3 Permeability

For calculating the intermediate and final floating position, the permeability of spaces is defined as function of the space type and the three draughts (deepest subdivision draught A_s, partial subdivision draught A_p and light service draught A_l).

2.10.4 Special requirements concerning passenger ship stability

In addition to the probabilistic approach as formulated above, passenger ships have to withstand damages which are defined by their location, length and depth. These requirements are to be regarded as an additional *deterministic* safety concept.

2

Ship Safety
Roll Motions in Waves

3 Ship Roll Motions in Waves: General Aspects

Potentially dangerous situations for ships operating in waves can be caused by

- roll resonance,
- pure loss of stability in wave crest condition,
- parametric roll excitations,
- large pitch motions,
- loss of manoeuvring capability.

The vessel's reaction is to be distinguished in

- excessive rolling $\rightarrow$ large roll amplitudes and/or large roll accelerations (Note: large roll angles do not necessarily result in large roll accelerations and vice versa),
- capsizing (worst case scenario),
- bow (stern) submergence $\rightarrow$ green water on deck,
- bow and/or stern slamming $\rightarrow$ impulsive pressure loads on ship structure,
- propeller racing,
- shift of loads on board,
- broaching.

Measures to increase ship safety with respect to its roll motions in waves are:

- minimization of GZ-curve fluctuation in waves $\curvearrowright$ design: hull form,
- avoidance of too small (minimum defined by IMO) and too large (!) GM values in operation ($\rightarrow$ loading condition), the latter generally resulting in high roll accelerations,

- application of roll damping devices like bilge keels and/or anti-roll tank ↷ design,
- avoidance of roll resonance through appropriate GM value ↷ operation: loading condition,
- minimization of excitations during voyage ↷ operation: course and speed,
- avoidance of shift of loads on-board ↷ operation: ensure secure lashing.

4 Environment: Wave Models

4.0.5 Regular Waves

Regular, sinusoidal waves can be described by:

- wave length $\lambda = \frac{2\pi g}{\omega^2} = \frac{gT^2}{2\pi}$
- wave height $H = 2 \cdot \overline{\zeta}$ with wave amplitude $\overline{\zeta}$
- wave slope amplitude $\overline{\vartheta} = \vartheta_{max} = k \cdot \overline{\zeta}$
- wave period $T = \frac{2\pi}{\omega} = \sqrt{\frac{2\pi\lambda}{g}}$
- wave frequency $\omega(\Omega) = \frac{2\pi}{T} = \sqrt{\frac{2\pi g}{\lambda}}$
- wavenumber $k = \frac{2\pi}{\lambda} = \frac{\omega^2}{g}$
- phase velocity $c = \frac{\lambda}{T} = \frac{\lambda \cdot \omega}{2\pi} = \frac{g}{\omega} = \frac{\omega}{k} = \sqrt{\frac{g}{k}} = \sqrt{\frac{g\lambda}{2\pi}}$

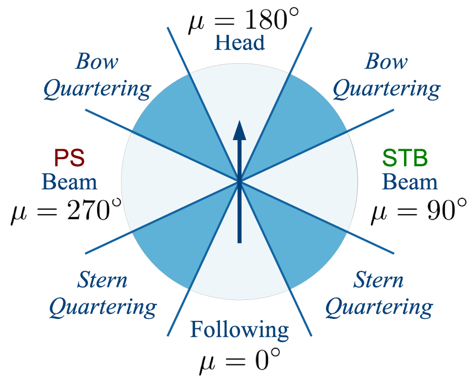

Figure 11: Wave direction

4.0.6 Oblique Sea: Transformation of Waves into Ship Coordinate System

Ship at speed v in waves, course with an angle μ, ($\mu = 0$ ↷ following sea, $\mu = 90$ ↷ beam sea, $\mu = 180$ ↷ head sea) relative to the dominant wave direction in a long crested seaway: frequency of encounter ω_e results from the transformation of the waves into a ship related coordinate system.

$$\omega_e = \omega - k \cdot v \cdot \cos\mu = \omega - \frac{\omega^2}{g} \cdot v \cdot \cos\mu$$

For distinct situations: head and following sea, the dimensionless frequency ratio becomes, see Figure 12:

$$\frac{v}{g} \cdot \omega_e = \begin{cases} \frac{v}{g} \cdot \omega + \left(\frac{v}{g} \cdot \omega\right)^2 & \mu = 180° = \text{head sea} \\ \frac{v}{g} \cdot \omega - \left(\frac{v}{g} \cdot \omega\right)^2 & \mu = 0° \text{ following sea and } c > v \\ \left(\frac{v}{g} \cdot \omega\right)^2 - \frac{v}{g} \cdot \omega & \mu = 0° \text{ following sea and } c < v \end{cases}$$

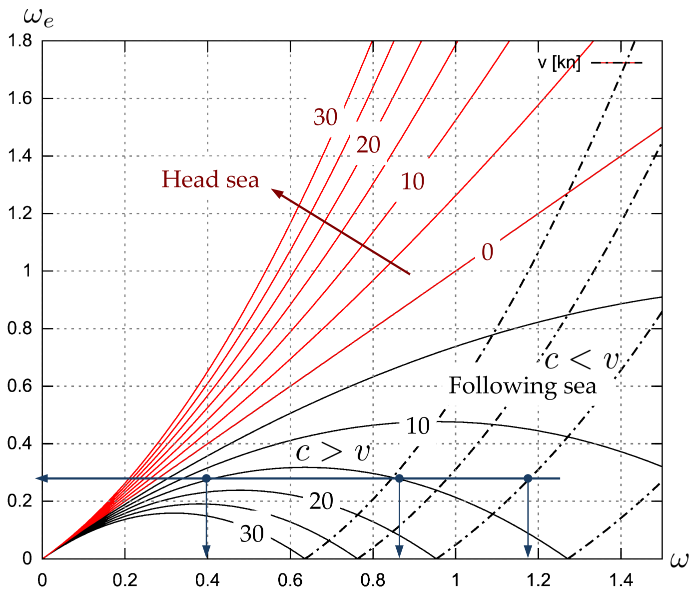

Figure 12: Relation of frequency of encounter ω_e and wave frequency ω with $\mu = 0$

4.0.7 Oblique Sea: Transformation of Waves into World Coordinate System

Ship at speed v in waves, course with an angle μ relative to the (dominant) wave direction with a frequency of encounter ω_e yields wave frequency ω in a fixed coordinate system. See Figure 13 for the combination of v, μ and wave period T resulting in potentially dangerous situations causing parametric roll excitation ($\rightarrow$ 4.3).

- Ship in head sea or long waves from aft or stern quartering sea.
  Note: $\omega \rightarrow \omega_e$ for $\mu \rightarrow \pm 90° = \pm\pi/2$

$$\omega = \frac{g - \sqrt{g^2 - 4gv\omega_e \cos\mu}}{2v\cos\mu}$$

- Ship in short waves from aft or stern quartering sea:

$$\omega = \frac{g + \sqrt{g^2 - 4gv\omega_e \cos\mu}}{2v\cos\mu}$$

4.0.8 Irregular Waves: Sea Spectra Models

For unidirectional waves (all waves in one direction) the *long crested* wave spectrum can be expressed by a spectral density distribution $S(\omega)$ to describe the seaway with the spectral energy $\rho \cdot g \cdot m_0$ and wave amplitude $\overline{\zeta}$:

$$m_0 = \int_0^\infty S(\omega)\, d\omega = \frac{1}{2}\sum_{n=1}^{N} \overline{\zeta}_n{}^2$$

From this definition the following characteristics can be derived:

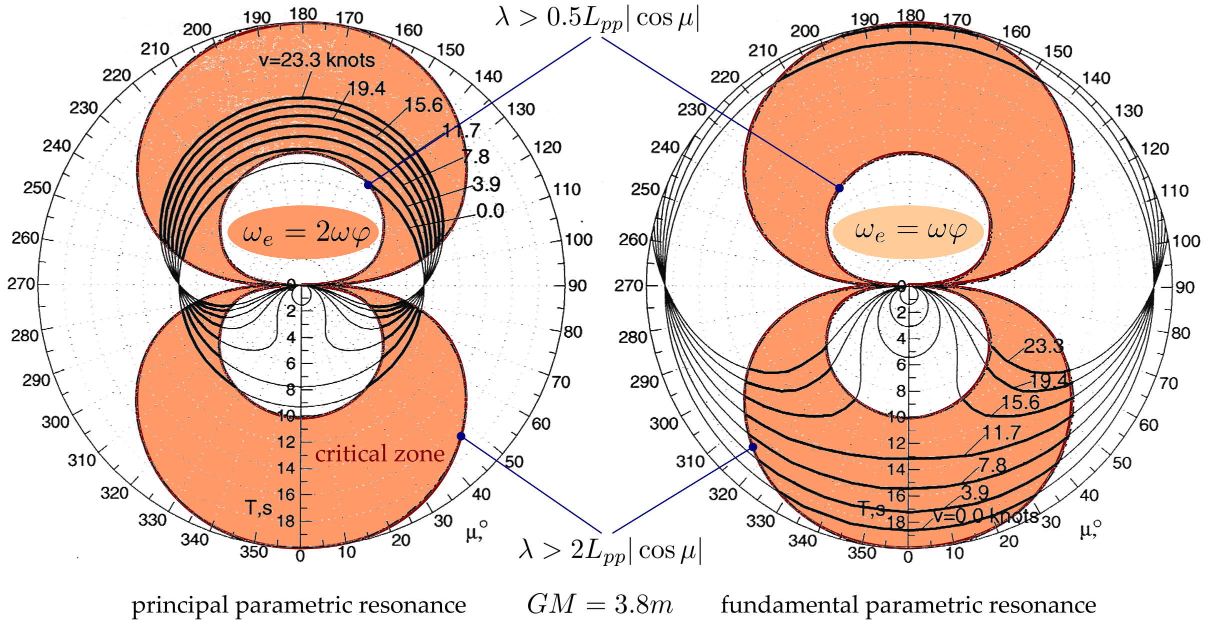

Figure 13: Combination of ship speed, course to waves μ yielding critical frequency ratios

- standard deviation of wave amplitudes: $\sqrt{m_0}$
- mean wave height of all waves $\overline{H} = \sqrt{2m_0}$
- realistic assumption: wave heights follow a Rayleigh distribution yields the *significant wave height* (average of the highest one third of all waves) $\overline{H}_{1/3} = 4 \cdot \sqrt{m_0}$
- wave height (average of the highest one tenth of all waves) $\overline{H}_{1/10} = 5.1 \cdot \sqrt{m_0}$

4.0.9 Pierson-Moskowitz Spectrum

Spectrum for fully developed sea, defined by significant wave height $\overline{H}_{1/3}$:

$$S(\omega) = \frac{a}{\omega^5} \cdot \exp\left[-\frac{b}{\overline{H}^2_{1/3} \cdot \omega^4}\right]$$

$$a = 0.78\ [m^2 s^{-4}] \quad \text{and} \quad b = 3.136\ [m^2 s^{-4}] \quad \curvearrowright S(\omega)_{max} \curvearrowright \omega_{\text{peak}} = \frac{1.258}{\sqrt{\overline{H}_{1/3}}}$$

Defined by wind velocity 19.5 m above sea level ($U_{19.5}$):

$$S(\omega) = \frac{\alpha g^2}{\omega^5} \cdot \exp\left[-\beta \left(\frac{g}{U_{19.5} \cdot \omega}\right)^4\right]$$

$$\alpha = 8.1 \cdot 10^{-3} \quad \text{and} \quad \beta = 0.74 \quad \curvearrowright S(\omega)_{max} \curvearrowright \omega_{\text{peak}} = \frac{0.877g}{U_{19.5}}$$

$$m_0 = 2.74 \cdot 10^{-3} \cdot \frac{(U_{19.5})^4}{g^2} \curvearrowright \overline{H}_{1/3} = 0.21 \cdot \frac{(U_{19.5})^2}{g}$$

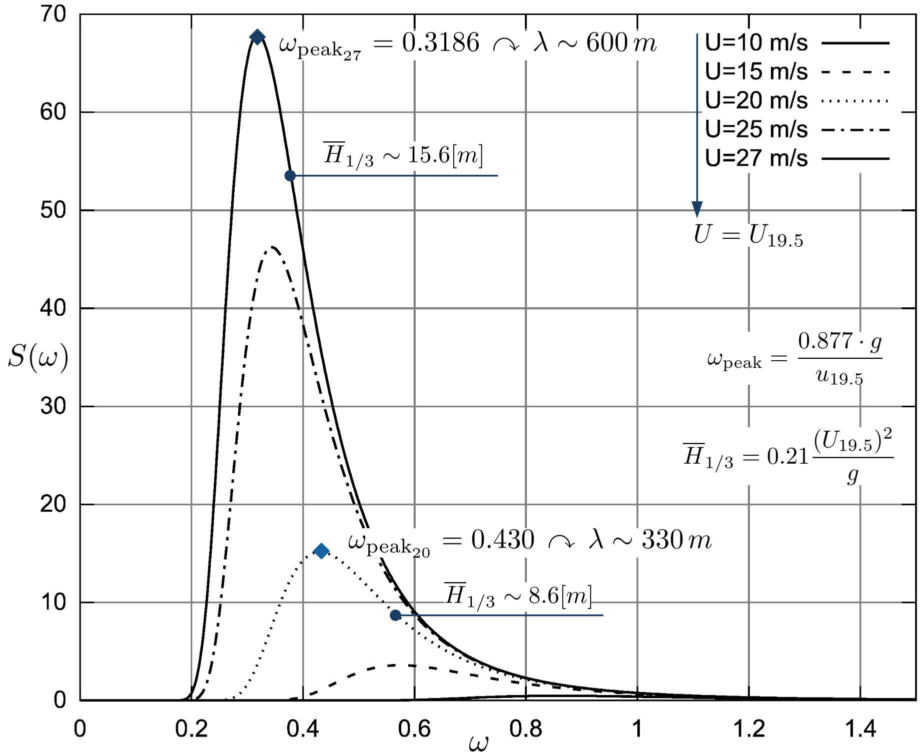

Figure 14: Pierson-Moskowitz spectrum as function of wind speed $U_{19.5}$

4.0.10 JONSWAP Spectrum

Spectrum definition of the "**Jo**int **N**orth **S**ea **W**ave **O**bservation **P**roject" (JONSWAP) with a fetch F, the distance from the lee shore:

$$\begin{aligned}
S(\omega) =& \frac{\alpha g^2}{\omega^5} \cdot \exp\left[-\frac{5}{4}\left(\frac{\omega_p}{\omega}\right)^4\right] \cdot 3.3^r \\
r =& \exp\left[-\frac{(\omega-\omega_p)^2}{2\sigma^2\omega_p^2}\right] \quad \text{with} \quad \sigma = \begin{cases} 0.07 & \omega \leq \omega_p \\ 0.09 & \omega > \omega_p \end{cases} \\
\alpha =& 0.076\left(\frac{U_{10}^2}{F \cdot g}\right)^{0.22} \quad \text{and} \quad S(\omega)_{max} \curvearrowright \omega_{\text{peak}} = 22\left(\frac{g^2}{F \cdot U_{10}}\right)^{1/3}
\end{aligned}$$

4.1 Free Roll Motion: Ship Roll Eigenfrequency

The differential equation for free roll motions neglecting damping simply yields

$$I' \cdot \ddot{\varphi} \quad + \quad \underbrace{\Delta \cdot g \cdot GZ(\varphi)}_{\text{non-linear restoring moment!}} \quad = 0$$

$$\text{with} \quad I' = k'^2 \cdot \Delta \quad (k' \approx 0.4 \cdot B) \quad \curvearrowright \quad \ddot{\varphi} = -\frac{g}{k'^2} \cdot GZ(\varphi)$$

Approximate solution, based on wall-sided-formula and series expansion of trigonometric functions:

$$GZ(\varphi) \sim \underbrace{GM \cdot \varphi}_{\text{linear term}} - \underbrace{\frac{GM}{6} \cdot \varphi^3 + \frac{BM}{3} \cdot \varphi^3}_{\text{non-linear term}} = GM \cdot (\varphi + c \cdot \varphi^3)$$

$$\text{with} \qquad c = \frac{3 \cdot \frac{BM}{GM} - 1}{6}$$

Special case $c = 0$: *linear restoring moment* $GZ = GM \cdot \varphi$ (only relevant for small roll angle φ)

$$\ddot{\varphi} + \frac{g}{k'^2} \cdot GM \cdot \varphi = 0 \quad \curvearrowright \quad \varphi = \overline{\varphi} \cdot \sin(\omega_\varphi \cdot t - \alpha)$$

$$\curvearrowright \quad \text{roll eigenfrequency} \quad \omega_\varphi = \sqrt{\frac{g \cdot GM}{k'^2}}$$

$$\curvearrowright \quad \text{roll natural period} \quad T_\varphi = \frac{2\pi k'}{\sqrt{g \cdot GM}}$$

General case$c \neq 0$: *non-linear restoring moment* $GZ = f(\varphi)$

$$\ddot{\varphi} + \frac{g}{k'^2} \cdot GM \cdot (\varphi + c \cdot \varphi^3) = 0$$

$$\curvearrowright \quad \left(\frac{T_\varphi}{T}\right)^2 = \left(\frac{\omega}{\omega_\varphi}\right)^2 = 1 + \frac{3}{4} \cdot c \cdot \overline{\varphi}^2 \quad \text{with} \quad \omega_\varphi \text{ the roll eigenfrequency for } c = 0$$

Solution: roll frequency ω is depending on the roll amplitude: $\omega = f(\overline{\varphi})$.

4.2 Regular Beam Waves: Ship Response

Forced vibrations due to beam waves, wave frequency Ω:

$$I' \cdot \ddot{\varphi} + \Delta \cdot g \cdot GZ(\varphi) = \Delta \cdot g \cdot GM \cdot \vartheta_{max} \cdot \cos \Omega t$$

Special case $c = 0$: *linear restoring moment* $GZ(\varphi) = GM \cdot \varphi$ ($\rightarrow$ only relevant for small roll angle φ)

$$k'^2 \cdot \ddot{\varphi} + g \cdot GM \cdot \varphi = g \cdot GM \cdot \vartheta_{max} \cdot \cos \Omega t \quad \curvearrowright \quad \varphi = \overline{\varphi}(\cos \Omega t - \varepsilon)$$

$$\overline{\varphi} = \frac{\omega_\varphi^2 \cdot \vartheta_{max}}{|\omega_\varphi^2 - \Omega^2|} = \frac{\vartheta_{max}}{|1 - (\frac{\Omega}{\omega_\varphi})^2|}$$

With damping proportional to roll velocity $\dot{\varphi}$

$$I' \cdot \ddot{\varphi} + W_\varphi \cdot \dot{\varphi} + \Delta \cdot g \cdot GZ(\varphi) = \Delta \cdot g \cdot GM \cdot \vartheta_{max} \cdot \cos \Omega t$$

$$\overline{\varphi} = \frac{\vartheta_{max}}{\sqrt{\left(1 - (\frac{\Omega}{\omega_\varphi})^2\right)^2 + \left(2D \cdot \frac{\Omega}{\omega_\varphi}\right)^2}} \quad \text{with} \quad D = \frac{W_\varphi}{\Delta \cdot k'^2 \cdot \omega_\varphi}$$

General case $c \neq 0$: *non-linear restoring moment* $GZ = f(\varphi)$

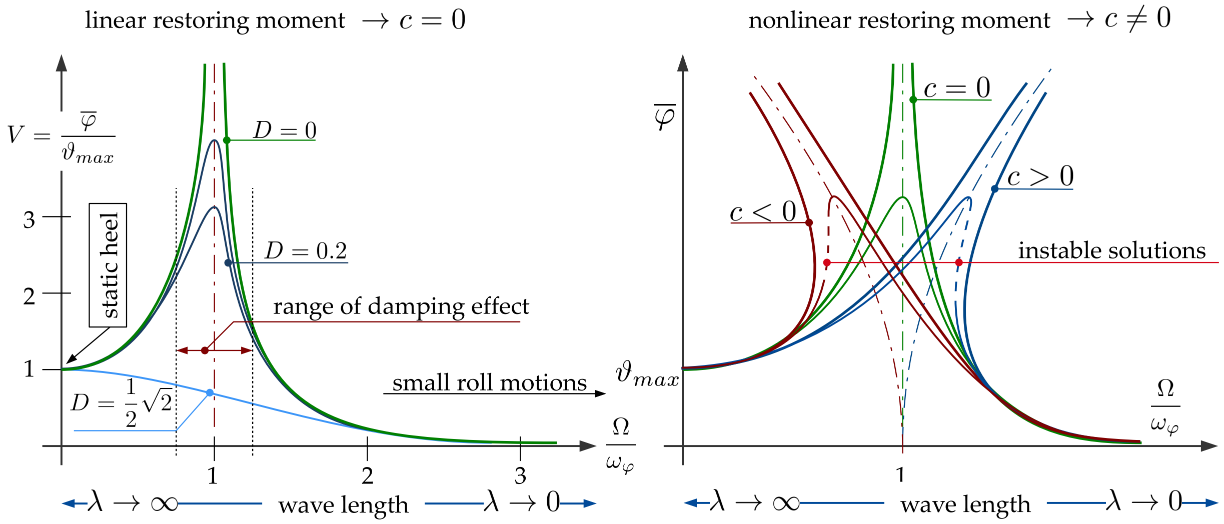

Figure 15: Ship forced roll motions: linear ↔ non-linear restoring moment

$$\left(\frac{\Omega}{\omega_\varphi}\right)^2 = 1 + \frac{3}{4} \cdot c \cdot \overline{\varphi}^2 \mp \frac{\vartheta_{max}}{\overline{\varphi}}$$

The solution of this differential equation, depending on the value of c is shown in Figure 15. In case of $c \neq 0$ the behavior of the vessel is given by the upper or lower part of the corresponding graph.

4.3 Parametric Roll Excitation: Ship in Longitudinal Waves

In case of waves running in ship's longitudinal direction: $\mu = 0°, 180°$, and a hull form symmetrically shaped with respect to center line, parametric roll excitations (no external excitation) exists caused by a restoring moment being a function of time as $GZ = f(\varphi, t)$:

$$I' \cdot \ddot{\varphi} + W_\varphi \cdot \dot{\varphi} + \underbrace{\Delta \cdot g \cdot GZ(\varphi, t)}_{\text{restoring moment } f(t)!} = \underbrace{0}_{\text{no external excitation!}}$$

Right side of above equation equals zero as head or following sea is assumed. Assuming a linear restoring moment (fully unrealistic for real hull forms and larger heeling angles!) but $GM = f(t)$ and also neglecting damping yields Mathieu's differential equation:

$$GZ(\varphi, t) \curvearrowright GM(t) \cdot \varphi = (GM_0 + \delta GM \cdot \sin \Omega t) \cdot \varphi$$

$$\cdots \downarrow$$

$$\ddot{\varphi} + \omega_\varphi^2 \cdot \left(1 + \frac{\delta GM}{GM_0} \cdot \sin \Omega t\right) \cdot \varphi = 0$$

This differential equation has stable ($\varphi \to 0$) and unstable ($\varphi \to \infty$) solutions for $\overline{\varphi}$, see Figure 17. Two frequency ratios which are specially problematic, see also Figure 13:

1. principle parametric resonance $\omega_e = 2 \cdot \omega\varphi$

2. fundamental parametric resonance $\omega_e = \omega\varphi$

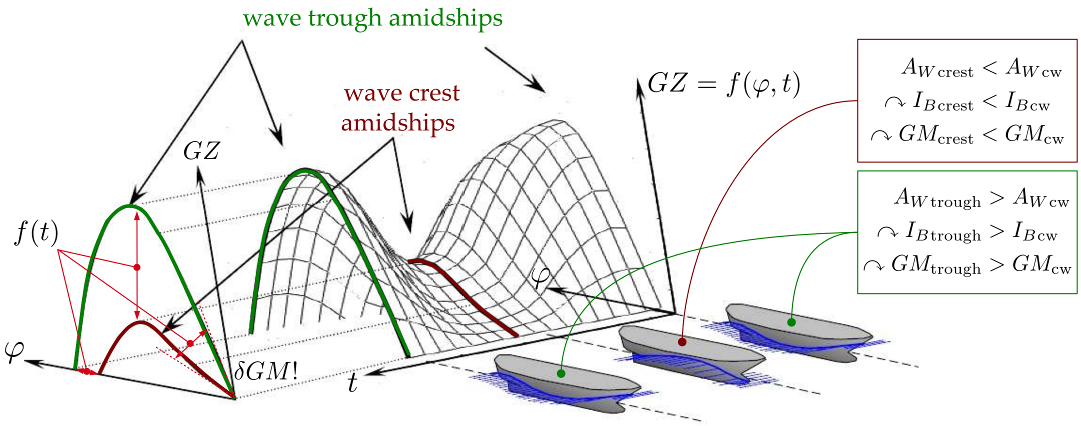

Figure 16: Righting arm fluctuations in longitudinal waves

Parametric roll excitations resulting in large roll amplitudes $\overline{\varphi}$ and most likely large roll excitations $\ddot{\varphi}$ specially occur if

- noteworthy GM fluctuations (δGM) exist in head or following sea $\rightarrow$ ship hull form
- roll eigenfrequency ω_φ and wave frequency Ω result in either principle parametric resonance or fundamental parametric resonance $\rightarrow$ ship loading condition ($KG \rightarrow GM \rightarrow \omega_\varphi$), course to waves μ and ship speed v,
- pitch natural period (T_{pitch}) is half of roll natural period (T_{roll}) $\rightarrow$ ship loading condition ($KG \rightarrow GM \rightarrow \omega_\varphi$), course to waves μ and ship speed v, see Figure 13 and 17.

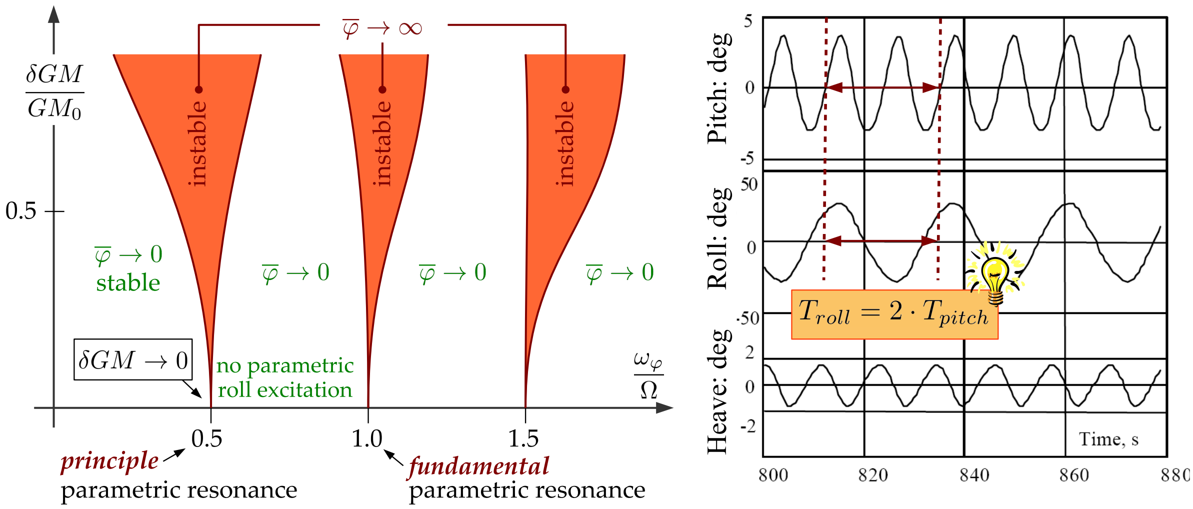

Figure 17: left: Stable and unstable solutions of Mathieu's differential equation – right: Ship motions showing problematic ratio of pitch and roll periods

4.4 Ship Response in Beam Sea but Irregular Waves

Under realistic sea conditions (irregular waves) the roll motion is random. With the roll response amplitude operator (RAO) $Y_\varphi(\omega)$ the roll spectrum yields:

$$S_\varphi(\omega) = S(\omega) \cdot Y_\varphi(\omega)^2$$

$$\int_0^\infty S_\varphi(\omega)\, d\omega = m_{0\varphi}$$

Realistic assumption: roll amplitudes follow a Rayleigh probability density function

$$P(\overline{\varphi}) = \frac{\overline{\varphi}}{m_{0\varphi}} \cdot \exp\left[-\frac{\overline{\varphi}^2}{2m_{0\varphi}}\right]$$

yields e.g. the *significant roll amplitude* (average of the highest one third of all amplitude) $\overline{\varphi}_{1/3} = 4 \cdot \sqrt{m_0}$, the average of the highest one tenth of all amplitudes $\overline{\varphi}_{1/10} = 5.1 \cdot \sqrt{m_0}$ and the cumulative distribution function

$$CDF = 1 - \exp\left[-\frac{\overline{\varphi}^2}{2m_{0\varphi}}\right]$$

4.5 Ship Response in Head or Following Sea (Irregular Waves)

Sea spectrum $S(\omega)$ to be transformed into sea encounter spectrum $S(\omega_e) = S(\omega) \cdot \frac{d\omega}{d\omega_e}$ results in narrow band spectral density distribution for following sea! Maximum (∞) at $\omega_e = 0.25 \cdot g/v$ $\curvearrowright \omega = 0.5 \cdot g/v$.

Potentially large roll amplitudes to be expected if:

ship hull form with noteworthy GM and GZ fluctuations in longitudinal waves

ship speed vs. wave height : ship speed v that sea state $\omega_{\text{peak}} = \frac{1.258}{\sqrt{\overline{H}_{1/3}}} \approx \omega_e = 0.25 \cdot g/v$

ship length vs. wave length : wave length $\lambda \approx$ ship length $\curvearrowright Fn = 0.2$

parametric resonance : principle ($\omega_e = 2\omega_\varphi$) and fundamental ($\omega_e = \omega_\varphi$) parametric resonance condition $\curvearrowright GM = f(k', \overline{H}_{1/3})$ and $GM = f(k', L)$.

4.6 Ship Response under any Sea Conditions

General approach: ship response under any wave direction μ and at any ship speed v: polar diagrams, see Figure 18. Left diagram: Blume criterion (→4.9) with effect of hull form modification but wave conditions constant, middle diagram: Blume criterion with effect of wave length variation but same hull form, right diagram: roll angle above threshold with effect of loading condition but same hull form and same sea state.

4.7 Roll Motions in 3 DOF

Nonlinear differential equation according to Söding, Kröger taking into account roll (φ), pitch (ϑ) and heave (z) motions:

$$I_x \cdot \ddot{\varphi} + d_l \cdot \dot{\varphi} + d_q \cdot \dot{\varphi}|\dot{\varphi}| + RM = M_{\text{dyn}} + M_{\text{Wind}}$$

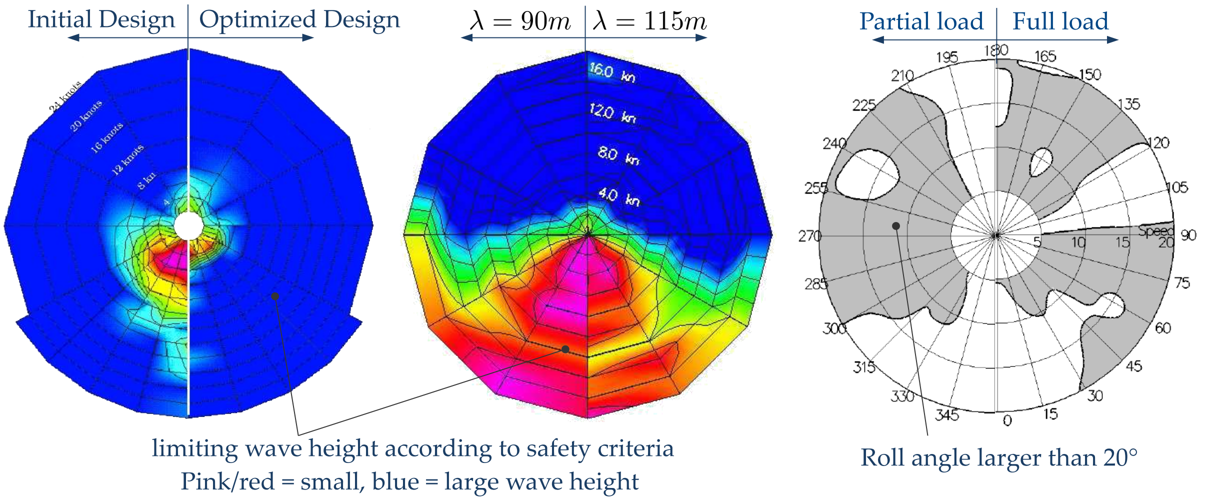

Figure 18: Polar Diagrams – Ship response taking sea conditions, varying speed and heading to any wave direction into account

With the restoring moment

$$RM = \int_L (g - \ddot{z} - x \cdot \ddot{\vartheta})(\rho \cdot A(x) \cdot KN(x) - \mu(x) \cdot KG(x) \cdot \sin\varphi)\, dx$$

and

$$\int_L A(x) \cdot KN(x)\, dx = KN \cdot \nabla \qquad \int_L \mu(x)\, dx = m = \rho\nabla \qquad \int_L \mu(x) \cdot KG(x)\, dx = KG \cdot m$$

yields

$$\begin{aligned} RM =& (g - \ddot{z}) \cdot m \cdot \underbrace{(KN - KG \sin\varphi)}_{GZ(\varphi,T,t)} \quad \text{[with average draft } T \text{ and trim } t\text{]} \\ & -\ddot{\vartheta} \int_L KN(x) \cdot x \cdot \rho \cdot A(x)\, dx + \ddot{\vartheta} \int_L KG(x) \cdot x \cdot \mu(x) \cdot \sin\varphi\, dx \end{aligned}$$

With

$$\int_L KG(x) \cdot x \cdot \mu(x)\, dx = I_{xz} \quad \text{and} \quad \underbrace{\int_L KN(x) \cdot x \cdot \rho \cdot A(x)\, dx}_{\text{to be neglected}} \ll I_{xz} \cdot \sin\varphi$$

finally yields for the differential equation

$$\ddot{\varphi} = \frac{M_{\text{dyn}} + M_{\text{Wind}} - d_l \cdot \dot{\varphi} - d_q \cdot \dot{\varphi}|\dot{\varphi}| - (g - \ddot{z}) \cdot m \cdot GZ(\varphi, T, t) - \ddot{\vartheta} \cdot I_{xz} \cdot \sin\varphi}{I_x}$$

4.8 Roll Damping

Damping of ship roll motions due to:

- wave generation,

- hull skin friction,
- generation of vertexes e.g. through bilge keels,
- anti-roll tanks.

According to Blume with ω_φ the roll eigenfrequency and $\frac{\varphi_{\text{stat}}}{\varphi_{\text{res}}}$ from measurements:

$$d_l = \frac{m \cdot g \cdot GM}{\omega_\varphi} \cdot \underbrace{\left(\frac{\varphi_{\text{stat}}}{\varphi_{\text{res}}}\right)\Bigg|_{\varphi \to 0}}_{f(B/T, c_B, F_n, \varphi_{\text{res}})}$$

$$dq = \frac{3\pi}{4}\left[\frac{m \cdot g \cdot GM}{\omega_\varphi} \cdot \underbrace{\left(\frac{\varphi_{\text{stat}}}{\varphi_{\text{res}}}\right)\Bigg|_{20^\circ}}_{f(B/T, c_B, F_n)} - d_l\right] + \frac{M_{BK}}{\omega_\varphi \cdot \overline{\varphi}^2}$$

Damping moment M_{BK} due to bilge keels:

$$M_{BK} = \rho \cdot \underbrace{b_{BK} \cdot l_{BK}}_{=f(\text{design})} \cdot \underbrace{J}_{=f(r,B)} \cdot \underbrace{r^3}_{\sim[T^2+(0.5B)^2]^{\frac{3}{2}}} \cdot \underbrace{c_D}_{=f(J,r,B,\varphi_A)} \cdot \dot{\varphi}^2$$

4.9 Blume Stability Criterion – IS-Code Form Factor

Approach: with respect to righting arm curve in calm water condition (!): evaluation of $\overline{E}_R$ and variance s through model tests or simulations. $\overline{E}_R$ is the averaged remaining area under the GZ curve, in case of capsizing $E_R = 0$. $\overline{E}_R$ to be regarded as measure for safety against capsizing.

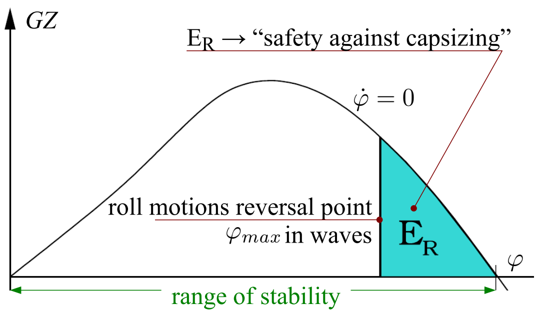

Figure 19: GZ curve: Blume stability criterion

Blume criterion: $[\overline{E}_R - 3 \cdot s] > 0$. As $\overline{E}_R$ is very much depending on the hull form and loading condition it yields: $\curvearrowright GM_{0\text{limit}} \curvearrowright KG_{\max} \curvearrowright$ minimum GZ curve: $GZ = KN - KG \cdot \sin\varphi$.

Blume's findings: Minimum GZ curve for same safety level against capsizing varies much with ship hull form and loading condition! Proposal: Form factor $C \uparrow$ for favourable ship hull form, $C \downarrow$ for unfavourable ship hull form to be applied in stability criteria.

$$C = \frac{T \cdot D'}{B^2} \cdot \sqrt{\frac{T}{KG}} \cdot \frac{c_B}{c_{WP}} \cdot \sqrt{\frac{100}{L}}$$

4.10 IMO IS-Code – From Physics to Regulations – a Critical Review

The criteria formulated in the IS-Code to be fulfilled for practically all vessels are shown in Figure 20. Additionally the ability of a vessel to withstand the combined effects of beam wind

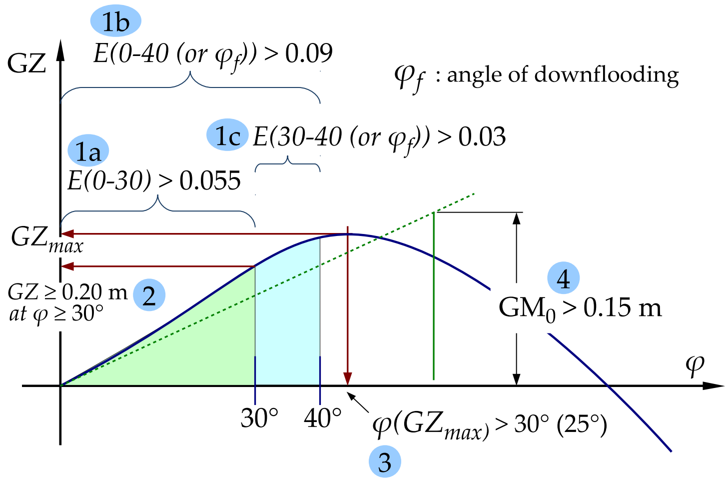

Figure 20: IMO IS-Code: General intact stability criteria for all ships

and rolling is to be demonstrated for which the weather criterion is formulated based on the calm water GZ curve.

Contrary to the risk based approach implemented in SOLAS, the IS-Code is purely based on a deterministic approach. Major arguments for the urgent need to reformulate the IS-Code taking more the meanwhile developed state of knowledge into account are:

- Ships operating fully compliant with respect to the actual IS-code can encounter dramatic situations leading to major damages to life and property or even the vessel's total loss.
- Except for containerships greater than 100 m (with form factor C as shown above), the utmost important effect of waves on ship motions and therefore on the vessel's safety is considered only by far too simplified models (statistics on required area under calm water righting arm curve for old ships not reflecting actual ship form developments, the weather criterion).
- The influence of the actual detailed ship hull form on the vessel's safety is only considered based on few integral values which are not sufficient to really judge upon the vessel's safety in a severe sea state.
- The criteria formulated on the calm water GZ curve are constant values for all ships independent from e.g. size and type (→ hull form characteristics). Hint: except for the German Navy (BV 1030-1), no intact stability code is known taking the impact of the specific ship hull form on the righting arm fluctuations in waves into account.
- Apart from a minimum GM requirement, a maximum GM requirement is regarded necessary to avoid large roll accelerations deteriorating the safety of people and property on-board.
- A deterministic approach is generally not capable to rationally evaluate the operational risk of a specific ship design under the large variety of loading and sea state conditions to be encountered in the ship's life.

List of Symbols and Acronyms

For SI-units holds:

Dimension symbol	Unit symbol
L	m
M	kg
T	s

∇	L^3	displaced volume
∇_0	L^3	displaced volume at initial floating condition: before damage
∇_R	L^3	displaced volume of object in final floating position after damage
∇_T	L^3	displaced volume of intact object in final floating position after damage
α	–	angle between axis x and principal coordinate axis x' of water plane after damage
Δ	M	Displacement ($\Delta = \nabla \cdot \rho$)
ΔGM	L	change of metacentric height: here due to a damage ($GM_0 - GM_R$)
ΔI_B	L^4	change of transverse water plane moment of inertia ($I_T - I_0$!)
ΔKB	L	change of vertical centroid of displaced volume: here due to a damage ($KB_0 - KB_R$)
Δp	–	partial survivability
Δx	L	shift of water plane coordinate system in longitudinal direction $CF_T \rightarrow CF_R$
Δy	L	shift of water plane coordinate system in transverse direction $CF_T \rightarrow CF_R$
δGM	L	change of metacentric height due to longitudinal waves (crest $\leftrightarrow$ trough condition)
δt	L	change of trim
ζ	–	dimensionless bottom damage penetration height: $\zeta = h/D$
$\overline{\zeta}$	L	wave amplitude
η	–	dimensionless damage length, measured in ship longitudinal direction: $\eta = l/L$
η_{50}	–	mean value dimensionless damage length according to statistic
ϑ	–	wave slope
$\ddot{\vartheta}$	$1/T^2$	pitch acceleration
κ_v	–	volumetric permeability ($0 \leq \kappa_v \leq 1$)
κ_a	–	area permeability ($0 \leq \kappa_a \leq 1$)
λ	L	wave length
$\mu(x)$	M/L	mass per unit length at longitudinal position x
μ	–	ship heading relative to waves (following sea: 0°, beam sea: ±90°, head sea: 180°)
ξ	–	dimensionless damage location in ship longitudinal direction
ρ	M/L^3	density of fluid, not further specified
τ	–	dimensionless side damage penetration depth: $\tau = t/B$
φ	–	angle of rotation about x-axis (ξ-axis) – heel
$\overline{\varphi}$	–	roll amplitude
φ_f	–	angle of downflooding
$\dot{\varphi}$	$1/T$	roll velocity
$\ddot{\varphi}$	$1/T^2$	roll acceleration
ϕ_e	–	list angle after damage (equilibrium floating position)
ω, Ω	$1/T$	wave frequency
ω_φ	$1/T$	ship roll eigenfrequency
ω_e	$1/T$	frequency of encounter: $\omega_e = \omega - k \cdot v \cdot \cos\mu = \omega - \frac{\omega^2}{g} \cdot v \cdot \cos\mu$
ω_{peak}	$1/T$	wave frequency at maximum $S(\omega)$ value, see Figure 14
A	–	IMO SOLAS attained index ($A \geq R$)
$A(x)$	L^2	section area at longitudinal position x

A_l	$-$	IMO SOLAS attained index at light service draft
A_p	$-$	IMO SOLAS attained index at partial subdivision draft
A_s	$-$	IMO SOLAS attained index at deepest subdivision draft
$A_{WL}(T)$	L^2	water plane area as function of draft
$a(x)$	L	floodable length, freeboard criterion
a_T	L^2	water plane area of flooded compartment (group) after damage
a_1	$-$	distance of vectors representing $\nabla_0 \cdot \rho \cdot g$ and $\nabla_T \cdot \rho \cdot g$ in final floating position
a_2	$-$	distance of vectors representing $\nabla_0 \cdot \rho \cdot g$ and $\kappa_v v_T \cdot \rho \cdot g$ in final floating position
B	ML/T^2	buoyancy force
B	$-$	symbol for centroid of buoyancy
B	L	ship moulded breadth
BM_0	L	transverse metacentric radius before damage
BM_R	L	transverse metacentric radius after damage
b	L	distance between ship hull and longitudinal bulkhead
$b(x)$	L	floodable length, stability criterion
b_{BK}	L	height of bilge keel profile
C	$-$	form factor as defined in IS-Code Chapter 4 – Special Criteria for Certain Types of Ships – 4.9 Containerships greater than 100 m
CDF		cumulative distribution function
CF	$-$	water plane area centroid, center of floatation
CF_T	$-$	water plane area centroid of intact object in final floating position
CF_R	$-$	water plane area centroid of damaged object in final floating position
c	L/T	wave phase velocity
c_B	$-$	block coefficient
c_D	$-$	resistance coefficient (here of bilge keels)
c_M	$-$	(midship) section area coefficient
c_{WP}	$-$	water plane area coefficient
D	L	ship moulded depth
D	$-$	dimensionless roll damping coefficient
D'	L	corrected ship moulded depth for C-factor calculation in IS Code
d_l	ML^2/T	linear roll damping coefficient
d_q	ML^2	quadratic roll damping coefficient
$\overline{E}_R$	L	Blume stability criterion: averaged remaining area under GZ-curve
F	$-$	factor of subdivision in deterministic damage safety approach
FB_{min}	L	minimum freeboard
F_n	$-$	Froude number ($F_n = v/\sqrt{g \cdot L}$)
FB_R	L	minimum freeboard after damage
GM_0	L	initial metacentric height before damage
$GM_{0\text{limit}}$	L	minimum metacentric height before damage
GM_0	L	average metacentric height in waves, $GM_0 \neq GM_{CW}$!
GM_{CW}	L	metacentric height in calm water condition: no waves
GM_R	L	metacentric height after damage
$GZ(\varphi)$	L	righting arm as function of heeling angle
GZ_{max}	L	maximum righting arm
$GZ_R(\varphi)$	L	righting arm as function of heeling angle after damage
H	L	wave height $(2 \cdot \overline{\zeta})$
$\overline{H}_{1/10}$	L	average of one tenth highest waves
$\overline{H}_{1/3}$	L	significant wave height: average of one third highest waves
h	L	penetration height in case of bottom damage

$I_{B_{min}}$	L^4	minimum moment of inertia of water plane in final floating position after damage ($I_{x'}$) about principle coordinate axis x'
I_{B_R}	L^4	minimum moment of inertia of water plane in final floating position after damage (I_x) about translated x-axis through CF_R
I_{B_T}	L^4	minimum moment of inertia of water plane of intact object in final floating position after damage
$I_{L_{max}}$	L^4	maximum moment of inertia of water plane in final floating position after damage ($I_{y'}$) about principle coordinate axis y'
I_{L_R}	L^4	maximum moment of inertia of water plane in final floating position after damage (I_y) about translated y-axis through CF_R
I_{L_T}	L^4	maximum moment of inertia of water plane of intact object in final floating position after damage
I_{Z_R}	L^4	product moment of water plane in final floating position after damage (I_{xy})
I'	ML^2	mass moment of inertia about ship longitudinal axis in roll motions: including hydrodynamic mass
i_T	L^4	transverse moment of inertia of water plane of flooded compartment (group) after damage with respect to midship axis x_0
i_{B_T}	L^4	transverse moment of inertia of water plane of flooded compartment (group) after damage with respect to a longitudinal axis through centroid water plane of flooded compartment (group)
i_{L_T}	L^4	longitudinal moment of inertia of water plane of flooded compartment (group) after damage with respect to a transverse axis through centroid water plane of flooded compartment (group)
i_{Z_T}	L^4	product moment of water plane of flooded compartment (group) after damage with respect to the axes through the centroid of water plane of flooded compartment (group)
I_x	ML^2	mass moment of inertia about ship longitudinal axis
I_{xz}	ML^2	mass product moment of inertia
IMO		International Maritime Organization
IS-Code		IMO Intact Stability Code
J	$-$	factor to express transverse flow velocity increase in bilge region due to roll motions
KB_0	L	vertical centroid of buoyancy before damage
KB_R	L	vertical centroid of buoyancy after damage
KB_T	L	vertical centroid of buoyancy of intact object but in final floating position after damage
kb_0	L	vertical centroid of volume of flooded compartment (group) before damage
kb_T	L	vertical centroid of volume of flooded compartment (group) in final floating position after damage
KG	L	vertical distance of G from K, position of vertical centroid of mass
$KG(x)$	L	vertical centroid of mass per unit length at longitudinal position x
$KG_{\max}$	L	maximum permissible vertical position of centroid of mass
$KN(x)$	L	transverse location of centroid of displaced unit volume (area) at longitudinal position x
k	$1/L$	wave number
k'	L	roll radius of gyration: $I' = k'^2 \cdot \Delta,\ k' \approx 0.4 \cdot B$
L	L	ship length, not further specified
LCB	L	longitudinal centre of buoyancy
LCF	L	longitudinal centre of floatation, centroid of water plane area
l	L	damage length
l	L	length of compartment (group), bounded by watertight transverse bulkheads, measured in ship longitudinal direction

l_{BK}	L	length of bilge keel profile
M_{BK}	ML^2/T^2	damping moment due to bilge keels in roll motions
M_{dyn}	ML^2/T^2	moment about x-axis due to waves, sway and yaw motions
M_{heel}	ML^2/T^2	heeling moment, not further specified
$M_{Passenger}$	ML^2/T^2	heeling moment due to crowding of people
$M_{Survivalcraft}$	ML^2/T^2	heeling moment due to launching of survival crafts
M_{Wind}	ML^2/T^2	heeling moment due to wind
m_0	L^2	integral value of sea spectral density distribution
m_{0_φ}	L^2	integral value of roll spectral density distribution
P	$-$	probability, here survivability of damaged vessel
$p(\xi,\eta)$	$-$	two parameter probability density function for damages
$p(\xi,\eta,\tau)$	$-$	three parameter probability density function for damages
R	$-$	IMO SOLAS required index ($R \leq A$)
RM	ML^2/T^2	restoring moment
r	L	damage length in case of ramming: measured from FP backwards
r	L	distance from longitudinally oriented roll axis to bilge keel, $r \approx \sqrt{T^2 + (0.5B)^2}$
rcb	L	distance of collision bulkhead from FP
SOLAS		IMO International Convention for the **S**afety **o**f **L**ife **a**t **S**ea
$S(\omega)$	L^2T	sea spectrum, see Figure 14
$S\varphi(\omega)$	L^2T	roll spectrum
s	-	variance
s	-	probability of survival after damage, factor to p in SOLAS
T	L	moulded draft
T	T	wave period ($\frac{2\pi}{\omega}$)
T_0	L	moulded draft, initial floating position, before damage
T_φ, T_{roll}	T	natural period of roll (e.g. reversal point port $\rightarrow$ starboard $\rightarrow$ reversal point port)
T_ϑ, T_{pitch}	T	natural period of pitch
T_{AP}	L	moulded draft at aft perpendicular
T_{FP}	L	moulded draft at forward perpendicular
t	$-$	time variable
t	L	penetration depth in case of side damage
t	L	trim ($T_{FP} - T_{AP}$)
$U_{19.5}$	L/T	wind velocity at 19.5 m above the water surface
v	L/T	ship speed
v_0	L^3	volume of flooded compartment (group), with upper boundary: water plane before damage has occurred
v_T	L^3	volume of flooded compartment (group), with upper boundary: final water plane after damage has occurred
W_φ	ML^2/T	roll damping coefficient
x	$-$	longitudinal coordinate axis through CF_R parallel to x_0
x_0	$-$	longitudinal coordinate axis through CF_T (center line axis)
x'	$-$	principal longitudinal coordinate axis of water plane in final floating position
x_C	L	centroid of water plane area of flooded compartment (group) in longitudinal direction, measured from y_0 axis
y	$-$	transverse coordinate axis through CF_R parallel to y_0
y_0	$-$	transverse coordinate axis through CF_T
y'	$-$	principal transverse coordinate axis of water plane in final floating position
y_C	L	centroid of water plane area of flooded compartment (group) in transverse direction, measured from x_0 axis
$\ddot{z}$	L/T^2	acceleration in vertical direction at longitudinal position $L_{pp}/2$

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